Non-cooperative Bargaining and Collusion Formation Through Communication Networks*

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Abstract

Many organizations rely on employees to supervise each other and perform their duties. In this paper, I model repeated games in which peer supervision structures can lead to equilibria where agents monitor and punish lack of effort. However, as efforts and punishments are costly, employees may deviate to a less costly equilibrium, resulting in department corruption. The paper models equilibrium selection as a bargaining process through a personal connection network, as corruption attempts cannot be public. A random initiator attempts to gain enough peer support to deviate to a new equilibrium. To be realistic, a unanimous consensus is not required.

The study finds that collusion is less likely with sparser personal connections. An algorithm is developed to identify critical players and links facilitating collusion, allowing policymakers to better control corruption through regulating personal connections or designing reward and punishment systems. This research offers insights into anti-corruption, anti-trust, firm management, political bargaining, social movements, and revolutions, particularly in cases where principals struggle to contract punishment after coalition formation

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1 Introduction

Collusion imposes a big problem for all kinds of organizations. This paper studies how individuals bargain for collusion through a communication network and how to optimally stop it. For example, consider a tech company owned by a boss with limited technical knowledge. The boss relies on a team of skilled employees to develop and produce the company's main product. Suppose the tech company is generating significant profits, with the majority of these profits going to the boss, while the employees who possess the technological expertise receive only a fraction of their wages. This imbalance in profit distribution may incentivize the employees to consider establishing their own company, develop a similar product, and secure the entire market share, ultimately leading to greater profits for themselves.

However, organizing such a collusion among employees is no simple feat. Due to the sensitive nature of their plan, discussions cannot be held openly, forcing the employees to rely on their personal connections and engage in a chain of persuasion. The challenge for the boss, then, is to design a system of rewards and punishments that effectively discourages the formation of a collusion, regardless of the initiator's identity or the employees they attempt to persuade.

This paper aims to explore the conditions under which a successful collusion may arise, identify the central player in organizing such a collusion, and determine whether it is possible to deter it entirely. By developing a systematic approach to analyze the collusion game within finite networks, this study introduces a novel centrality measure that assesses the significance of different positions in the network, ultimately shedding light on the dynamics of employee collusion in the tech industry.

Here is a brief outline of the model. We start with an organization operating on a principal-optimal equilibrium. In such an equilibrium, agents suffer from losses that they want to get rid of. So, in each period, agents can talk to each other and plan for a joint deviation to a new equilibrium (i.e., collusion). Every period, a random agent becomes the initiator and proposes a new equilibrium, and if there is enough support (at least m out of n total agents agree to jointly deviate), the entire department moves to the new equilibrium. Otherwise, the agents adhere to the principal's plan. This equilibrium selection mechanism resembles a vote, with an exogenously given passing threshold. The mechanism follows the bargaining literature (Rubinstein, 1982a; Binmore et al., 1986; Chatterjee et al., 1993; Battaglini, 2021). In reality, a collusion does not require everyone's consent. Many collusion proceed even with strong objections. So, in this paper, I allow the success threshold m to vary and study how it affects the bargaining process.

Here is a preview of the results. In the case when all agents can directly communicate with one another (unlimited communication), (1) collusion can be stopped if and only if the success threshold (m) is higher than a critical value and (2) the deviation can be stopped much easier if the agents are less connected.

The vulnerability of the unlimited communication network comes from the following intuition: if one agent rejects the initiator's offer to collude, the initiator could ask other agents to collude, without the knowledge of the first one. If the initiator succeeds, those who vote in favor of collusion get a favorable equilibrium, whereas the others get an undesirable outcome. [1] So, if collusion will occur even without the honest agent joining it, this agent would be better off by agreeing to collusion instead. Through backward induction, the reward for whistleblowers will not be effective unless everyone expects that enough people will reject the offer to collude.

If the agents are less connected, then each agent can ask for a more favorable new equilibrium in the bargaining stage. This is because for the initiator to reach the rest of the agents, the initiator needs her direct acquaintances to forward the message to the agents she is not connected to. Thus, the direct acquaintances can ask for a larger share of the collusion benefit, which makes the initiator less likely to get enough support for collusion to happen.

By the end of this paper paper, I also discuss an alternative bargaining model in which the agents can commit. Commitment gives agents much stronger ability to deviate. Thus, the results are quite different from the "seeking support" model.

In this paper, I first characterize the communication network that is hardest for the agents to collude. When the principal wants the equilibrium to sustain the largest "costs" for the agents, a ring network is the best choice. "Costs" means the agents' lost total welfare when they cannot deviate from the principal optimal equilibrium to the agent optimal equilibrium. In the ring, each agent only connects to two people, on the person's left and right. With this ring structure, for all thresholds $m \leq 3$, all deviation can be deterred no matter how many agents there are in the department. If largest "costs" are not required, then a star network can further reduce the critical threshold to 2. However, the central player in the star network needs to share a large fraction of the total benefit with the principal,and the principal shares only a small fraction of the benefit with the other agents.

Besides the limiting cases, I also solve the bargaining game under any arbitrary communication network structure. The solution could be obtained by an algorithm of solving a system of linear equations. Using this algorithm, I obtained a new network centrality measure which I call Bargaining power. The bargaining power of an agent is measured by the largest number of colleagues that the agent can convince to collude. The larger the number, the higher the bargaining power is. Using this algorithm, principals can identify the agents who are most likely to initiate the collusion and thus give him or her special attention. The

[1]For instance, if a whole department with the exception of one person is corrupt, the corrupt agents can fake evidence and falsely accuse the honest agent for misbehavior.

principal could also try to limit the communication among some critical agents to further reduce the risk of collusion.

Limiting communication to deter joint deviation has been used in the real world. For instance, government official who attempts corruption might not know all the people in the court, the police department, or the press, so joint collusion may be detected by outside agents. Thus, corruption is less likely to occur if agents fear the act of corruption could be detected. Similarly, some large companies impose strict communication among critical employees. For instance, to prevent corruption of a firm's cashier, accountant, and warehouse managers, the employer usually adopts the practice called separation of duties, which means different people have different roles. Thus, no single agent can fake evidence and cover up illegal actions. Attempts to establish a personal connection would also be punishable, resulting in violators being dismissed from their position.

However, as well as deterring collusion, communication control can also be used in a bad way. A dictator may enjoy the benefit of exploiting the people by putting them in a peer supervision structure. To maintain such a desirable equilibrium, the dictator usually imposes strict limitations on communication. Methods include news censorship, cover-up, and other information manipulation. Attempts of upheaval are cracked down. Thus, a revolution would become unlikely. Though all the rewards and punishments come from the people and are inefficient, people under a dictatorship would not be able to deviate from the undesirable equilibrium and overthrow the dictator.

Similarly, some large companies stop their frontline workers from forming a labor union by limiting communication among them. Managers get rewards if they report on the flyers that attempt to unionize the workers. Companies hire union-busting services to crack down on union groups on social media. These are attempts to directly cut the communication links among the workers.

The theory shows how these kinds of control can be used for both efficient control and also for exploitation. When the latter happens we hope that the theory can be used to develop more elaborate structures to prevent the unwanted exploitation.

This paper has the following contributions: First, it suggests the importance of stopping agents from renegotiating a welfare-improving new equilibrium because such a deviation may harm the principal or damage the social welfare. I propose a new way to model joint deviation, instead of using equilibrium refinement. Then, I study two paradigms of how players may reach an agreement in each case and deter Pareto-improving collusion. Finally, I find that limiting communication among the players can significantly improve the robustness of the effort provision equilibrium, and this conclusion has many real-world applications.

In the following subsection, I discuss the related literature. In section 2, I set up the base model and derive a benchmark. In sections 3, I introduce the bargaining process mechanism solving for the extreme cases and then introduce the algorithm to deal with arbitrary communication network. Finally, in section 4, I conclude this paper with some extensions and a discussion of other real-world applications of my paper.

1.1 Related literature

In infinite repeated games, we frequently see equilibria which majority of people get undesired payoff (just as the firm example at the beginning of the introduction). Such equilibria are usually sustained by peer supervision and mutual punishment among players. However, the stability of the peer supervision structure has been questioned long before my work. Farrell and Maskin (1989) made the following comments about potential renegotiation: "unless players somehow cut the line of communication, it seems possible that they can renegotiate after the game begins, that they will not follow a mutually-unpleasant subgame-equilibrium path when there is a Pareto dominating alternative available, even if they agree to do so when the game begins." To address this concern, the authors develop the renegotiation proof refinement on the subgame perfect equilibrium. The equilibrium selection process in my paper is also related to equilibrium refinement such as the concept of coalition proof equilibrium (Bernheim et al. (1987))

Building on their idea, I provide a network bargaining model of the renegotiation process. I relax the assumption that a joint deviation requires unanimous consent among players. Instead, I assume that deviations may occur if a sufficient number of players agrees. A communication network structure is also introduced since it plays a critical role in the renegotiation process.

This paper is also related to the literature on monitoring and community enforcement. The earliest work is Akerlof (1976). More recently Basu (2018) extends the original idea from Akerlof and built a peer supervision model that directly inspired this paper. Most of this literature on community enforcement focuses on the threat of withdrawal of the cooperation and does not consider costly punishments (Kandori, 1992, Ellison, 1994, Kranton, 1996, Wolitzky, 2013, Ali and Miller, 2014). A few papers do allow punishment, mostly focusing on enforcers' incentive to carry out punishment (Dixit, 2011, Masten and Prüfer, 2014, Levine and Modica 2016, Aldashev and Zanarone, 2017, Acemoglu and Wolitzky, 2019). Another study branch concerns the upper bound of welfare in repeated games that is achievable by punishment and rewards (Acemoglu and Wolitzky, 2017, 2018). These authors find relatively tight bounds in repeated games with an arbitrary number of individuals. The main difference between these papers and mine is that they focus on reinforcing the efficient outcome, whereas I study reinforcing the seemingly undesirable equilibrium so that an organization can be resistant to collusion (joint deviation). In addition, I study costly punishment and collusion as an endogenous equilibrium selection process.

The renegotiation (bargaining) mechanism described in this paper is borrowed from the bargaining and coalition formation literature (Rubinstein, 1982a; Binmore et al., 1986; Chatterjee et al., 1993; Battaglini, 2021; Ray, 2007). In this paper, I incorporate a communication network into the bargaining process and find that the network positions of individual players significantly change their bargaining power. Based on this finding, I characterize the most robust communication network against collusion and derive more general results for arbitrary communication networks.

My model is different from the bilateral network bargaining games in Manea (2011); Nguyen (2015). They model the pairwise agreement (buyers and sellers) among players in the network and characterize the stable equilibrium outcome. In my paper, however, I use the voting mechanism to model an overall equilibrium selection process. It is fundamentally different from pairwise agreement and is thus suitable for a different set of real-world applications.

The model of this paper is similar to that in Roemer (1985). He describes a game between the Czar and Lenin. Lenin tries to overthrow the Czar by proposing a new distribution rule and forming a larger coalition. The Czar, on the other hand, implements a set of punishments on those who join the coalition, thus reducing the likelihood of the revolution. If the revolution is successful, then no punishment can be implemented. The Czar is like the principal in my model, and Lenin is like the initiator. The major difference between the two models is that my model is dynamic, and the new distribution has to be supported by an equilibrium. In addition, I add a dynamic communication network to the negotiation process. Holding a different network position gives an individual different bargaining power and thus significantly affects that individual's payoff in the outcome. Grossman (1991) extend the model of Roemer (1985) and assume the benefit of a successful revolution is shared only within the group of participants instead of the entire society; this assumption is a natural equilibrium result of my model. Later literature on revolution also studies the effect of communication cost on revolution (Little, 2016; Enikolopov et al., 2011): control communication to increase the cost of organizing collective action. My theory provides a microfoundation of the communication cost.

Finally, I recognize the potential connection between my work and network cooperative games. One may believe it is possible to map my negotiation outcome to a specific set of characteristic functions in transferable utility cooperative games. Then, whether an equilibrium is resistant to collusion can be similar to finding a core in a cooperative game. However, it is unclear how to do this, because my model imposes many restrictions on possible coalition configurations. The reward and punishment rules also restricts the distribution of payoffs.

2 The Base Model: No Communication

I begin with a base model in which communication among the players is not allowed. This model establishes a benchmark for the largest amount of total effort that is possible under community reinforcement. This is a perfect information finite-agent repeated game. Denote the set of players (or agents) as N. There are $n+1$ players in the game. Player zero is the principal, and players $1-n$ are the agents. I mainly focus on the cases in which $n \geq 2$, which means there are at least two agents, because peer supervision is not possible when there is only one agent. Denote the set of agents (excluding the principal) as I. In the rest of the paper, I will keep referring to the firm management problem. The principal is the owner of the firm; the agents are employees.

I assume the principal wants the agents to work as hard as possible. However, the agent cannot directly contract any punishment or reward based on the outcome, because, as I will assume later, the agents can collude and fake evidence to avoid any punishment or abuse the rewards. Thus, these policies are ineffective once some of the crucial agents collude. This is what happens in the firm example, when the critical players are the warehouse manager, the cashier, and the accountant. Another reason for no reward or punishment is that the cost of supervision is too high for the principal, so she will not observe if the agents slightly decrease the effort. So, in the base model, I keep the same assumption to establish a benchmark. The only thing that the principal can do is to choose an initial equilibrium for the agents through pre-job training or to set ground rules. Before the agents start to work, the principal tells each agent the other players' strategy and the corresponding best response. Thus, when the game starts, none of the agents has the incentive for unilateral deviation from the designated strategy. At period $t = 0$, the principal chooses the equilibrium for the agents to maximize the sum of discounted daily effort levels. Then the agents' repeated game starts from period $t = 1$.

In each period $t \geq 1$, there are two stages, similar to the morning and the afternoon of a working day. In the first stage of period $t \geq 1$, each agent $i \in I$ can simultaneously choose an effort level e_{it} and a vector of peer transfers.

In the firm example, exerting effort e_{it} reduces the utility of the agent by e_{it} in period t. The cost of effort comes from two channels: it is costly for the agents to (1) perform the daily tasks and (2) forgo the potential benefits of corruption. For instance, a cashier may defraud his employer and earn an extra 1000 dollars per day by inflating the purchasing price of inventory. So, in my setting, not being corrupt and not getting the extra income is labeled as the agent losing 1000 dollars' worth of utility in a day.

Let $\pi_{it} = (\pi_{i1t}, \pi_{i2t}, ..., \pi_{iNt})$ denote the peer transfers that agent i gives to the other agents 1, 2 ... N. Assume every $\pi_{ijt} \geq 0$ for all $i \in \mathcal{I}$, $j \in \mathcal{I}$, and $t \geq 0$. All the transfers are positive because the agents can only choose how much to give to the others, but they cannot choose how much they take from the others. Here, I assume $\pi_{iit} = 0$, because the transfer to oneself has no use. Denote the vector of effort $e_t = (e_{1t}, e_{2t}, ...e_{nt})$, and denote the set of transfers $\{\pi_{ijt}\}_{i,j\in\mathcal{I}}$ as $\pi(t)$. A transfer can be thought of as one player giving money to another, though in reality, the transfer can be more diverse: giving praise and glory, improving friendship, giving other kinds of favor, or anything that costs one player some effort and makes the receiver happier. To avoid the Ponzi scheme, I assume the daily transfer possible for each agent is bounded: $\pi_{ijt} \leq \overline{\kappa}$ for all i and all j, where $\overline{\kappa}$ is a large positive number.

In the second stage of period $t \geq 1$, all players observe π_{ijt} and e_{it} for all i and $j \in \mathcal{I}$, and then each player $i \in I$ simultaneously chooses a vector $\phi_{it} = (\phi_{i1t}, \phi_{i2t}, ..., \phi_{iit}, ..., \phi_{int})$ in which for all $i \neq j$, $\phi_{ijt} \in [0, 1]$, and for all i, j, and t. Here $\phi_{ijt} \in [0, 1]$ represents the punishment technology available to each player. Agents can choose the severity of the punishment, but the maximum punishment is normalized to 1. When $\phi_{ijt} = 0$, it means player i does not want to initiate a fight with player j. If $\phi_{ijt} \in (0, 1]$, it means player i initiates a punishment against player j and then, whether j wants to fight or not, both parties lose ϕ_{ijt} units of utility. In a firm, the punishment can be one agent criticizing another for not finishing the job or not following the rule. A more sever kind of punishment might involve monetary losses such as getting a fine or losing a bonus. The largest can be one agent reporting that another is committing crime, so the latter can be incarcerated. [2] I also assume the agent who punishes other agents loses utility for many reasons. For instance, she may suffer emotionally for inflicting harm to her peers. The main results in this paper still hold when the agent who punishes other agents does not suffer significantly more than they do. Denote the set of punishments as $\{\phi_{ijt}\}_{i\in\mathcal{I}\&j\in\mathcal{I}} = \phi_t$. Here, $\phi_t \in \Phi$ is the set of possible punishments in the second stage of each period t.

The effort benefits the principal, so she wants to choose an equilibrium in which the agents work as hard as possible. In order for the agents to maintain the effort, punishment is needed for peer supervision. Transfers are required to incentivize punishment. Furthermore, the transfers are especially important in the bargaining for joint deviation. In some cases, the initiator needs to promise "bribery transfers" to make another peer vote yes. I assume there is no discount from the first to the second stage of a period because this assumption provides a more straightforward characterization of the maximum sustainable effort $\overline{e_i} = n - 1$. The main results do not change if the agents discount between the first and second stages of the game.

Letting u_i denote the per period utility that agent i receives in period $t \geq 1$, with

[2] If one player is incarcerated or dismissed from the department, I assume there will be another identical agent taking the exact position of the departing agent in the next period, so the resulting game is similar to the infinite repeated game described here.

$u_i : \Pi \times \Phi \mapsto \mathbf{R}$, results in

$$u_{it}(\pi(t), \phi(t)) = -e_{it} - \sum_{j \in \mathcal{I}} \pi_{ijt} + \sum_{j \in \mathcal{I} \& j \neq i} \pi_{jit} - \sum_{j \in \mathcal{I} \& j \neq i} max\{\phi_{ijt}, \phi_{jit}\}$$

The first component is the loss from exerting effort. The second term is the sum of all transfers that player i gives away, and the third term is the sum of all transfers that player i receives from the other players. The last component is player i's loss from exerting or receiving punishment in the second stage. I further assume every individual has a δ discounted utility function

$$U_i = (1 - \delta) \sum_{t=1}^{\infty} \delta^t u_{it},$$

where u_i is the per period utility that agent i receives in period $t \geq 1$.

I assume that the entire game history is known to all agents in this game. For tractability, I focus on pure strategy equilibria.

The history of the first stage of period t is the collection of all player actions that happened before period t: $h_{1t} = \{\pi_1, \phi_1, ..., \pi_{t-1}, \phi_{t-1}\}$. Similarly, the history of the second stage of period t is the collection of all player actions that happened before day t plus all player actions in the first stage of day t: $h_{2t} = \{\pi_1, \phi_1, ..., \pi_{t-1}, \phi_{t-1}, \pi_t\}$. I distinguish between h_{1t} and h_{2t} because at the second stage of the day, all players can base their decision on what happened in the first stage of the day π_t.

Define a pure strategy of an individual $i \in \mathcal{I}$ as two functions, $s_{1,i} : h_{1t} \mapsto \pi_{it}$ and $s_{2,i} : h_{2t} \mapsto \phi_{it}$. Let $s_{1,i} \in S_1$ and $s_{2,i} \in S_2$ be the two sets of maps, and define $\mathcal{S} = \{S_1, S_2\}$ as the strategy set. This paper focuses only on pure strategy equilibria because the pure strategies alone can sustain the minmax payoff, which covers all the extreme cases that are interesting.

A complete game is defined as $\Gamma = \{I, \mathcal{S}, U, \Pi, \Phi\}$, where I is the set of players, $\mathcal{G}$ is the set of strategy, U is the payoff function, and Π and Φ are the action sets in the first and second stages of each period, respectively.

In period 0 (in which the agents do nothing), the principal chooses a subgame perfect equilibrium ($eq \in EQ$) to maximize the sum of discounted effort:

$$U_p(eq) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \sum_{i \in I} e_{it}. \tag{1}$$

In this model, the principal's welfare improves as each $\pi_{i,i}$ increases while the other variables remain fixed. However, effort is privately costly to the agents, making any equilibrium with a positive effort level undesirable for the agents. If I ignore the principal and look only at the agents, any equilibrium that has strictly positive $\pi_{i,i}$ in this model is Pareto dominated by another equilibrium that sets $\pi_{i,i} = 0$.

In this model, the principal is more like a designer of the game, because as the game starts she cannot affect the outcome anymore and has to rely on her design of the equilibrium to achieve her goal. In the base model I set up a benchmark on the highest payoff that the principal can possibly achieve when the agents cannot communicate. Then, I compare the results to those of alternative models in which the agents can communicate and jointly deviate to another equilibrium that may harm the principal. When communication is allowed, the principal must trade off the total effort against the stability of the supervision network. In general, less communication implies less effort, but the equilibrium is more robust against collusion.

2.1 Effort-provision and Corruption

In the basic model, a natural solution concept is the subgame perfect Nash equilibrium. This section establishes a benchmark for making players exert a positive amount of effort in the absence of coalition or collusion. First, I define the terms effort provision and corruption in the context of my paper.

Definition: An effort provision equilibrium is any equilibrium in which there exists at least one player $i \in \mathcal{I}$ who chooses effort $e_{i,t} > 0$ for a period $t \geq 0$. An agent exerts full effort if he is indifferent between exerting the effort or receiving the maximum punishment in a day. No equilibrium effort level can be larger than the full effort.

Definition: A corrupt equilibrium is any equilibrium in which all players $i \in \mathcal{I}$ choose effort $e_{i,t} = 0$ for all periods $t \geq 0$.

These two types of equilibria are the focus of this paper. The principal wants to maximize the sum of total effort, but putting the players into an effort provision equilibrium, as shown in lemma 1, causes the agents to lose utility; therefore, if they can coordinate a joint deviation, they will never exert any positive effort level.

Lemma 1. *Given the number of agents $n \geq 2$ and a discount factor δ greater than $\frac{1}{2}$, the largest sustainable daily effort level for each agent is $n - 1$.*

Proof of lemma 1: The proof can be directly derived from the folk theorem using a trigger strategy. When there are n agents in a department, each agent's minmax payoff in one period is $-(n-1)$. This is achieved by all the other agents punishing a target agent while he does not make any transfer or exert any effort. Thus, all effort levels smaller than $n - 1$ are individually rational and can be sustained in the repeated game. If the stage game utility is lower than $-(n-1)$, then the target agent would rather be punished by all the other agents. Thus, any stage game utility strictly lower than $-(n-1)$ is not sustainable. The details of such a subgame perfect Nash equilibrium are in the appendix A1.

Proposition 1. *When only the welfare of the agents is considered, every effort provision equilibrium is Pareto dominated by a corrupt equilibrium.*

The proof is straightforward. For any effort provision equilibrium, there is a corresponding corrupt equilibrium in which all the players use the same strategy except no player exerts any effort on the equilibrium path. Thus, some agents must have a higher utility in the corrupt equilibrium while the others are not worse off.

Based on this proposition, the effort provision equilibrium is stable only when the agents in the department cannot talk to each other and thus cannot form coalitions for deviation to a new equilibrium. If all the agents can jointly renegotiate and the principal cannot punish the deviation, then intuitively the agents can deviate to a Pareto improving corrupt equilibrium. In most real-world cases, agents should be able to talk to each other and negotiate a new equilibrium. For instance, if the workers of a firm share the same office space and can freely communicate with each other, such joint deviation to corruption is very likely. The following corollary is another way to illustrate this problem of instability.

Corollary 1. *If the agents can form a joint deviation without the consent of the principal* [3]*, then any effort provision equilibrium*[4] *is neither a coalition proof equilibrium (Bernheim et al. (1987)) nor a weakly renegotiation proof (Farrell and Maskin (1989)).*

The detailed proof is in the appendix A2.

However, in this paper, I find that if the re-negotiation process is explicitly modeled, then limiting the communication channels among the agents can significantly reduce the likelihood of a joint deviation (i.e., collusion).

First, I study a model of voting under full communication. In this game all the players can directly communicate with each other. If the agents choose the equilibrium by voting, when the threshold of passing the vote is sufficiently high, then the deviation to a corrupt equilibrium cannot form.

In the second model, suppose the principal fully controls the initial communication network among the agents. Then, the effort provision equilibrium becomes much more robust. I characterize the most robust communication network. In this model, a receiver has to be

[3]Because I assume the principal cannot punish or reward the agents after the game starts, it is reasonable to assume that the agents do not need the principal's agreement for deviation.

[4]The effort provision equilibrium is the subgame after the principal chooses the equilibrium.

contacted by the initiator so that the receiver can cast a vote, because otherwise, the receiver does not know there is a voting stage going on and so cannot vote. Thus, when there are fewer communication links, each agent can demand a more favorable equilibrium after the agent is approached by the initiator if the initiator needs the receiver to help transmit the message of voting to the other agents. Thus the initiator is less likely to benefit from collusion than in the unlimited communication case.

However, I realize that full control of communication is not always possible. Therefore, I also develop an algorithm to calculate the bargaining power of each agent when the communication network is exogenously given. The formation of the communication network is an involved process which I do not model. There are different costs and benefits for one player to keep a connection with another player. I assume the agents will be in an equilibrium communication network that takes all the costs and benefits into account (including the potential benefit from successful collusion). Using this algorithm, organizations such as firms or governments can check if a department is at risk of collusion. The algorithm can also help construct policy interventions that make collusion less likely.

3 Voting for Deviation

This section provides a model of voting for equilibrium selection. The idea is that when the agents are unsatisfied with the status quo, there is a probability that one of the agents becomes the initiator of collusion and proposes a new equilibrium to the other agents. If there is enough support for the deviation, then the department jointly deviates to the new equilibrium. Here, I assume unlimited communication among the agents so the initiator can directly contact each receiver. For instance, suppose an accountant finds a loophole in the firm's internal control system. If one of the two cashiers and one of the three warehouse managers agree to collude, they can stop supervising each other and jointly fake transaction records and make a sizeable illegal profit to share between them. [5] In the meantime, the firm's owner cannot find such a crime because the colluding party has hidden all the evidence. With some probability, the accountant may propose this plot to her colleagues, and if the threshold of success is reached, they will deviate to a new equilibrium. Similarly, if a cashier (or a warehouse manager) finds the same loophole first, the cashier can initiate collusion and the technical threshold of passing the deviation is the same.

To abstract away from the technical details for such collusion, I assume that all the agents follow an equilibrium selection rule: an initiator can propose a plan for the new equilibrium. If m out of n agents vote yes, the entire department follows the initiator's

[5] For instance, they can falsely claim there is severe water damage in one of the warehouses, so some of the items there have to be discarded. However, in reality the damage is minor and no items have to be discarded. The colluding agents sell the items and keep the profit to themselves.

proposal. Otherwise, they adhere to the principal's plan. The threshold m is exogenously given by the other technical details or related legislation that I will not model here.

The threshold m can be the majority rule, but sometimes more essential decisions may require $\frac{2}{3}$ of the voters to agree. In some other cases, a few players (e.g., leaders) may have higher weights than others, and thus, a proposal may be passed even when only a small fraction of players agree to it. In this paper, I study the effect of different levels of the threshold m so the result is relevant to many real-world cases.

The formal model of the voting procedure is the following: In period zero, the principal first chooses a subgame perfect equilibrium $eq_{default}$ for all the agents. From period 1 to infinity, all the agents start playing in this equilibrium.

Every day, with a small probability p_0[6], one agent is randomly chosen to be the initiator and she comes up with a plan of deviation. Then, the initiator chooses whether to start the voting stage and pass the deviation proposal. If not, the initiator remains silent, and all the players continue with the default equilibrium. [7]

When the voting stage starts, the vote of individual i is denoted as $v_i \in \{yes, no, NA\}$, where NA stands for not being able to vote because the initiator (denoted by subscript i) does not reach the player; NA has the same effect as voting no. Let EQ denote the entire set of subgame perfect equilibria in the repeated game after the voting stage. A specific element is the set denoted as eq. Define the equilibrium selection function of the principal (denoted by subscript i) as a mapping from the identity of the initiator (I) and the realized votes (V) to an equilibrium outcome:

$$eq_p : I \times V \to EQ.$$

Similarly, define the equilibrium selection function of the initiator as

$$eq_i : I \times V \to EQ.$$

The equilibrium selection function eq_i allows the initiator to propose favorable equilibria for those who vote yes; similarly, ep_p allows the principal to choose favorable equilibria for those who vote no. Such flexibility is crucial to the analysis.

The principal specifies the equilibrium selection function eq_p, so the agents understand that if they reject a deviation proposal they follow eq_p. The initiator observes the eq_p and then chooses whether to initiate the vote. If the vote begins, the initiator chooses the equilibrium selection function eq_i to best respond to the principal and calls the other agents one by one

[6] I start with small probability p_0 so that the possibility of deviation does not collapse all the incentives for punishment and reward before the deviation happens. In the extension, I will discuss the case when probability p_0 is large. However, that case does not differ much from the analysis here if collusion can be stopped, because if no one can initiate collusion, it does not matter how frequent the initiator arrives.

[7] Since coordinating the vote is costly, not everyone is willing to vote every day. Only someone with a preference shock that makes her willing to coordinate the vote will be the initiator.

to collect their votes. Each receiver, upon being called, votes either yes or no. This is called sequential voting. The main results are the same if all the agents vote together after the negotiation stage; therefore, I focus on sequential voting here.

A case in which the initiator chooses to be silent is equivalent to the case of no initiator being selected for the period. All the players continue with the default equilibrium $eq_{default}$. Finally, $eq_{default}$ has to be an equilibrium under any proposal of the initiator, as well as being consistent with eq_p when the deviation vote is rejected. If the initiator starts the voting stage, the initiator automatically votes yes, and then all the other players would know who the initiator is.[8]

In this section, I assume that there is no limit on communication. All the players are directly connected. In the next sections, I allow the principal to design the initial communication among the agents, and I study how such limited information helps increase the robustness of the effort provision equilibrium. Thus, any two agents can directly transmit information and money to anyone else. Also, all the communication about the voting stage is fully observed by all the agents. [9]

The voting stage ends when at least m yes votes are collected, or the initiator has contacted all votes but does not have enough positive votes. One may also consider a simultaneous voting mechanism; however, the sequential vote is identical to the simultaneous vote if the principal specifies the order of rewards given to the players who reject the vote. Therefore, in this paper, I consider only the sequential voting paradigm.

Assumption 1. *Assume that once a deviation proposal is passed, there will be no future initiator whether the current proposal is voted upon favorably or not.*

This assumption, used in network bargaining papers such as (Nguyen, 2015), is strong because it represents an extreme case where the initiator does not need to worry about her proposal being overturned by future initiators; therefore, she can promise "yes" voters more desirable equilibria than what she can do absent this assumption. The results derived represent an upper bound of the initiator's bargaining power. Consequently, if the principal can stop the deviation with this assumption, the principal can also stop all potential deviations without this assumption. [10]

[8] I assume that the players can freely pass around this information.

[9] For instance, the initiator can honestly tell each receiver what happened in the previous round of negotiation. Then after the voting stage ends, all colluding agents can tell the other agents what happened and what equilibrium they should play for future periods.

[10] If I change this assumption to allow a new voting stage if in the previous one the initiator fails to convince enough people to vote yes, and if the full effort is supported in the default equilibrium, then under the new assumption, the critical voting threshold for deterring collusion does not change. The idea is the following: the relaxed assumption makes a difference only when the first voting stage results in more votes against the collusion proposal than in its favor. One could worry that in the punishment stage, a new initiator may appear and initiate a new collusion proposal, so the punishment may not be credible. However, since full effort is supported in the default equilibrium, no player will be worse off than in the default equilibrium, and thus no one will have an incentive to collude in the punishment

Let $R \in pass, fail$ denote the result of the vote. If there are at least m players who vote yes, then all the players follow the equilibrium selection function of the initiator, and if there are insufficient "yes" votes, then the equilibrium selection function of the principal is followed. Assume all the agents have the same patience level δ, and they choose the strategy to maximize the discounted total welfare. Let $u_{i,t}(eq)$ denote the payoff to player i on day t when the equilibrium is eq. Let $U_i(eq) = \sum_{t=1}^{\infty} \delta^{t-1} u_{i,t}(eq)$ denote the utility of player i when the equilibrium in the repeated game turns out to be eq. By the Folk theorem, $u_i(eq) \geq -(n-1)$.[11] for all i and all eq.

I allow both the principal and the initiator to choose an equilibrium selection function instead of a single equilibrium because the former setting enables both parties to differentiate between the agents who vote against the proposal and those who vote in favor of it. Such differentiation is crucial for the successful deterrence of undesired deviation.

The principal's goal. Assume the principal still wants to maximize the discounted sum of effort of the agents. However, since the agents can now coordinate a joint deviation, the principal also cares about deterring collusion. For simplicity, I assume the principal has two goals: (1) stop collusion if possible and then (2) choose the equilibrium to maximize the sum of total effort. This twofold objective function is similar to assuming the principal incurs a large loss when collusion occurs. For the rest of the paper, I will focus on how to check if collusion will happen, since it is a critical intermediate step to solve the principal's problem.

It is worth pointing out that in the current model, I assume there is no communication cost, all agents are identical and prefer not to work, and the principal cannot exert any external punishment or reward before or after the collusion. However, all these assumptions can be relaxed. As I will show later, these assumptions mainly affect the resource constraints for both the principal and the initiator. In the later sections, I will show an algorithm that can be used to calculate how those factors change the possibility of collusion. However, I first focus on a benchmark model with the aforementioned restrictive assumptions and show how individuals' position in a communication network affects their bargaining power to negotiate collusion. In the following section, I solve the model and explore the conditions under which it is possible to deter collusion.

stage. So, the principal can still deter collusion using the same punishment rule. The problem is more complicated when the full effort is not supported in the default equilibrium. For this analysis it is necessary to discuss individual incentives for collusion in the punishment stage. This analysis can be done using the algorithm that I will describe later in this paper.

[11] This proof is in Appendix B1.

3.1 Solving the Equilibrium

In this model, I use the standard practice in the voting literature and assume that players would not choose weakly dominated strategies in the voting stage. This assumption will eliminate the equilibria in which the agents reject Pareto improving proposals because too many other players decide to vote no. The resulting equilibrium will thus be unique.

Lemma 2. *If a joint deviation vote is passed, no player exerts a strictly positive effort in the new equilibrium.*

The proof is similar to that of proposition 1. The details of the proof are in Appendix B1 Lemma 2 significantly reduces the set of proposals available to the initiator. This lemma also shows that it is important for the principal to know when and how to stop the joint deviation, because if a joint deviation ever occurs, the principal loses all the benefits from effort.

This section discusses how the principal may stop the joint deviation using conditional punishment and reward. The main result is the following proposition:

Proposition 2. *The joint deviation to no effort can be stopped if and only if*

$$m > \frac{n^2 + n - 1}{2n - 1}. \tag{2}$$

Also, if $m > \frac{n}{2} + 1$*, agents can exert full effort* $e_0 = n - 1$ *in the default equilibrium, yet joint deviation does not occur.*

The detailed proof is in Appendix B2. The intuition is that to stop the joint deviation (coalition), the principal must make at least $n - m + 1$ players vote no. On the other hand, the initiator needs to make precisely m players vote yes. Thus, the initiator could choose an equilibrium selection function to promise favorable equilibria to the "yes" voters and unfavorable equilibria to the "no" voters. So, if the deviation is passed, the "yes" voters get transfers from the "no" voters. Expecting the initiator's proposal, the principal would choose the opposite reward schedule: reward those who vote no, and such reward has to come from those who vote yes. If the number of players m is large, there would not be enough reward to incentivize enough people to vote yes; thus, corruption does not occur. On the other hand, if the number of players is small, the principal would have trouble incentivizing the players to reject the deviation (i.e., collusion). So, I can derive the cutoff value of m by comparing the disposable rewards. Notice that the proposition 2 also implies that deviation to corruption always happens when the majority voting rule is used. So, one may still question the stability of the effort provision equilibria in reality, because the majority rule is very commonly used. In the next section I report my findings that the principal can make the effort provision equilibrium more robust by limiting the communication among the agents.

3.2 Voting under Limited Communication

In the previous section I discussed how the agents endogenously choose the equilibrium when all the players are fully connected. Here, I provide a formal model of the voting stage under limited communication and characterize the upper bound for the robustness of communication against collusion. If the principal can limit the communication among agents, a critical receiver can shield the initiator from reaching the rest of the agents by rejecting the deviation proposal. Thus, the initiator needs to promise a very desirable equilibrium to the critical voter. As a result, the initiator may exhaust all the rewards before enough people vote in favor of the deviation proposal.

Revisiting the firm example, suppose all the critical employees work from home. They have to use a specialized communication application to exchange messages. Suppose their real-world identities are hidden, and workers do not previously know each other. Consequently, they are unlikely to meet each other offline and establish new connections. Thus, the principal can fully control the communication network among the workers. In this structure, if one worker wants to send a message to someone he is not connected to, he needs to ask other agents to be the mediator and forward the message to the target worker.[12] Thus, the intermediary agent can refuse to send a message if it contains a deviation proposal. Similarly, the intermediary can refuse any request for a private communication method such as phone numbers or email. If the principal can incentivize each receiver not to forward unallowed messages to other agents or establish private communication with other agents, then collusion will not occur. Limiting communication may be possible in other situations when it is costly for the agents to meet each other and establish communication without the mediation of a mutual acquaintance (call this unmediated communication), for instance, when agents are spatially separated and job duties are not published.

In this limited communication model, I assume the history of the voting stage is perfectly observed only by the agents who receive a message from the initiator. When the voting stage ends, all the other players then observe the voting result and the equilibrium to follow. This assumption implies that an agent cannot vote without being contacted by the initiator first, because the agent does not know there is voting.

Ideally, collusion never occurs if the principal eliminates all communication links. However, that is an unlikely scenario, because workers will not exert any effort without supervision. It is reasonable to assume that if one player can supervise another, then they must be linked through communication. Then, I define supervisors and supervisees.

Definition 1. *A player i is the supervisor of player j if and only if i can initiate a punishment against j. Also, player j is called the supervisee of player i. The set of supervisor and*

[12] For instance, if A connects to B and B connects to C while A is not connected to C, then A needs to ask B to forward the message to C.

supervisee is listed in an $n \times n$ square matrix $\mathbf{S}$. *An entry* $\mathbf{S}_{i,j} = 1$ *if* i *is the supervisor of* j*, and* $s_{i,j} = 0$ *if* i *is not the supervisor of* j*.* [13]

Think about the supervision network as specified by a contract. If punishment is not allowed in the labor contract, the player cannot carry out the penalty against another player; thus, the supervision network does not expand over time. Let $d_i(S)$ denote the number of agent i's supervisor given network S.

Then, I can define the communication network and the constraints on it.

Definition 2. *Let* $\mathbf{C}$ *be an* $n \times n$ *symmetric matrix that represents the communication network. If* $\mathbf{C}_{i,j} = 1$*, it means player* i *can directly send any message to player* j*. If* $\mathbf{C}_{i,j} = 0$*, then players* i *and* j *are not directly connected.*

The communication in this model is always bilateral. This is a realistic assumption: if two players are connected, one should be able to call the other and vice versa. Ideally, the principal wants the agents to have as little communication among them as possible. However, it is natural to assume that each supervisor needs to communicate with her supervisees to make monitoring and punishment possible. Thus, there exists the following constraint on the communication network:

Assumption 2. *For all* $i, j \in N$ *and* $i \neq j$*, where* N *is the set of players in the department, if* $s_{i,j} = 0$ *and* $s_{j,i} = 0$*, then* $\mathbf{C}_{i,j} = \mathbf{C}_{j,i} = 0$*; otherwise* $\mathbf{C}_{i,j} = \mathbf{C}_{j,i} = 1$*.*

Finally, I assume that all the agents' transfers must go through some communication paths. Therefore, there are no isolated agents and disconnected components. If a department has two separate components, they should be studied as two different departments, because the agents in the two components have no influence on one another.

The timing of the voting stage is the following: When the initiator calls a receiver and proposes a deviation plan, the receiver can choose between voting yes and voting no. If the receiver chooses yes, she also agrees to forward the message of the collusion plan to the other receivers. Thus, the initiator can message all the neighbors of the receiver in the communication network. Assume there is no cost to forward a message. If the receiver chooses to vote no, she also refuses to forward the message to other agents on behalf of the initiator. [14] Let i denote the initiator, and j denote the receiver. If j votes yes, then for all $\{r \in N | \mathbf{C}_{j,r} = 1\}$, $\mathbf{C}_{i,r} = 1$. It means the initiator can message agent j's neighbors. The communication network stays the same if the receiver j rejects the offer to collude.

[13] Because $\mathbf{S}$ is a directed graph, it does not need to be symmetric.

[14] I do not consider the cases in which the receiver votes no but agrees to forward the message and that in which the receiver votes yes but refuses to forward the message, because these strategies are inherently inconsistent. In the first case, forwarding the message facilitates the joint deviation, but voting no undermines the collusion. It is always weakly better for the receiver not to share the contact information if the receiver expects collusion to fail, or vote yes if the receiver expects collision to pass. The same reasoning applies to the second case.

An alternative interpretation of the extended communication network is that agent i asks agent j to give away her neighbors' contact information. This interpretation is the same as forwarding a message if establishing new direct communication has no cost. However, getting to know new people and earning their trust to discuss collusion is usually very costly. So, forwarding the message should be a more reasonable interpretation of the extended communication network.

The negotiation stage ends when the initiator has sent the message to all her neighbors (including those newly connected neighbors), but the collusion proposal fails to pass or whenever m "yes" votes are collected. Under this modified model, it is harder for the initiator to organize collusion because of the communication constraint. Each receiver has more power to reject the vote because the receiver can block the initiator from reaching other individuals if the receiver chooses to vote against the collusion proposal. I can then construct a measure of robustness for any supervision network using the following notations:

Definition 3. *Let $\underline{m}_i(\mathcal{S})$ be the largest voting cutoff such that if player i is the initiator and $\mathcal{S}$ is the supervision network, the deviation vote can be passed. The term $\underline{m}_i(\mathcal{S})$ is the bargaining power of player i in supervision network $\mathcal{S}$.*

Intuitively, m_i measures the number of "yes" votes the initiator can obtain given the supervision network when the principal chooses the equilibrium selection function eq_p to minimize this number. The larger the number m_i, the more bargaining power player i has, because a higher m_i means the agent i can get more people to accept her collusion proposal.

Definition 4. *Let $\underline{m}(\mathcal{S})$ denote the robustness of the supervision network $\mathcal{S}$. Then, $\underline{m}(\mathcal{S}) = \max_{i \in N}\{m_i(\mathcal{S}).\}$*

The robustness of a supervision network is determined by the player with the largest $\underline{m}_i$. If there is at least one initiator who can successfully coordinate a deviation, then the effort provision equilibrium vanishes with probability 1 as time goes to infinity. The principal has the same goal as before. Given the voting threshold m, the principal first chooses the supervision network S and equilibrium selection function eq_p to deter collusion and then maximizes the discounted sum of effort.

3.3 Network characterization

In this subsection, I solve the collusion formation game and characterize the robustness of each network. I start by providing a theoretical upper bound on how robust a communication network can be. Since both the robustness of a network $m(S)$ and the effort level in the default equilibrium depend on each other, I focus on the following two benchmarks: (1) What is the most robust equilibrium and supervision network when players can sustain the full effort? (2) What is the most robust equilibrium when full effort is not required? Full effort under limited communication is defined below.

Definition 5. *A full effort equilibrium is one in which on the equilibrium path, each player i can exert effort $e_0(i) = d_S(i) - \epsilon$ in the default equilibrium, for all $\epsilon > 0$.*

Here $d_S(i)$ is the number of supervisors of player i in network S. I am especially interested in the full effort equilibrium for two reasons: First, this type of equilibrium fully utilizes each individual's supervision. Second, if I assume the severity of punishment is independent of the number of supervisors that an agent has, then the effort levels are the same for all full effort equilibria. Yet, it is possible to find a unique, more robust supervision structure.

Reducing the effort level in the default equilibrium could further reduce each agent's incentive for a deviation. I am also interested in the most robust network without the full-effort constraint. In later sections, I separately characterize the most robust network structures with or without the constraint of full effort.

I show that a single ring supervision network is the most robust network under full effort. Using a star network further increases the robustness; however, the sustainable effort level decreases significantly to achieve the increment of robustness.

Definition 6. *A single ring supervision network $S \in \mathbf{S}$ is one where $s_{i,i+1} = 1$ and $s_{i+1,i} = 1$ for all $1 \leq i \leq n-1$, $s_{n,1} = s_{1,n} = 1$, and $s_{i,j} = 0$ otherwise (i.e., every player is both the supervisor and supervisee of her two neighbors).*

Then, I have the following propositions:

Proposition 3. *When the voting threshold for the deviation proposal to pass is m and full effort is required, the principal can stop joint deviation for all $m \geq 3$ and all $n \geq m$, using a single ring supervision network and a corresponding equilibrium selection function eq_p.* [15]

The proof is in Appendix C1. According to this proposition, no matter how large the department is, the principal could stop all deviation attempts as long as the voting threshold for the deviation proposal to pass (m) is no less than 3. Thus, for all $m \geq 3$, there is a corresponding full effort supervision network that can deter joint deviation.

The main difference between the fully connected communication network and the ring is that in the former one, even if one receiver votes no, the initiator can still contact the player "behind" this receiver, and thus the coalition may still form. When the coalition forms, the first receiver who votes no could suffer greatly. Thus, if the principal wants to stop collusion, she has to stop $n - m + 1$ voters from saying yes to the deviation proposal. Such reward may be too costly when m is small. However, in the ring, the initiator can contact only

[15] If $n = 2$ and $m = 2$, the deviation can be stopped. When $m = 1$, deviation always occurs.

her "neighbors". If the two neighbors do not agree to the deviation plan, they can stop the initiator from reaching the rest of the agents. Thus, I refer to the players who can shield the initiator from the rest of the department as the gatekeepers. They have a combined veto power, which means that if they reject the initiator's offer, then the deviation cannot occur. Thus, the gatekeepers can ask for large compensation to say yes to the deviation proposal. The bribery could be so large that the initiator is unable to credibly promise them they will obtain it even if the deviation proposal is passed. Anticipating this, the initiator keeps silent and stays in the effort provision equilibrium. The following two corollaries describe why the single ring supervision network is a good combination of robustness and full effort.

Corollary 2. *Given a size of department $n \geq 3$, the single ring supervision network is weakly more robust (i.e., has a weakly smaller $\underline{m}(\mathcal{S})$) than all supervision networks that can sustain full effort.*

Corollary 3. *For all supervision networks S with robustness $m(S) = 2$, the single ring supervision network (i.e., that in which a player is both the supervisor and supervisee of her two neighbors) generates the highest total daily effort level.*

The corollary 2 and corollary 3 can be proved together. The details are in Appendix C2. The two corollaries suggest that if the principal wants to maintain the full effort, the most robust supervision structure is the ring and the lowest bargaining power measure is $m = 2$. A natural result is that if the severity of punishment is independent of the number of supervisors and there is no change in the communication cost among the agents when the communication path gets longer, then for all $m \geq 3$, putting the agents in a ring supervision network is the best the principal can do.

However, the single ring is not the most robust network when full effort is not required. In an extreme case, the principal may face a threshold of $m = 2$, and if the principal is sufficiently averse to collusion, she may choose a star network, sacrificing the total effort that she can get.

Definition 7. *The most robust network S has a robustness measure $m(S) = 1$ (i.e., deviation can be stopped if $m = 2$).*

In the most robust networks it is not possible to deter deviation when the initiator can determine the equilibrium outcome ($m = 1$). See the following lemma:

Lemma 3. *For all $n \geq 2$ and $m = 1$, for all supervision networks S and all equilibrium selection functions of the principal eq_p, a deviation occurs almost surely as time goes to infinity.*

The proof is simple: When $m = 1$, the initiator always chooses the equilibrium in which all the other agents give the initiator the highest possible transfer every day. Then, for all $n \geq 2$, joint deviation always occurs because it is impossible to give all the players the maximum reward in the default equilibrium. There must be at least one player who would benefit from deviation; thus, deviation always occurs as time goes to infinity. So, the most robust supervision network can deter collusion only when $m = 2$.

Proposition 4. *Both full effort and no collusion cannot be achieved when $m = 2$ and $n \geq 3$.*

The proof is the following: when full effort is required, the reward pool that the initiator can use is just ϵ smaller than the reward pool of the principal. When $n \geq 3$, at least one player must be connected to at least two neighbors. This player can always promise the first receiver who votes yes a reward strictly smaller than r. In this case, one of the neighbors would accept the offer because the principal cannot promise both neighbors a reward of r.

Thus, to sustain a full effort, the agent with more than one neighbor needs to exert less effort and get a transfer from the others in the default equilibrium to reduce her incentive to coordinate deviation.

The most robust network to deter collusion is a star: there is a central player and all the other peripheral players connect only with the central player. However, the central player does not exert effort in the default equilibrium. This player gets large transfers from the peripheral players to reduce her incentive for deviation. The details of the star network and corresponding equilibrium selection functions are in Appendix C3.

The above propositions describe the trade-off between effort, supervision, and robustness. In general, when collusion is hard to form (i.e., when the voting threshold for the deviation proposal to pass, m, is large), the principal can introduce more peer supervision and thus induces more effort, yet collusion does not happen. When m is small, the principal needs to restrict the amount of supervision, resulting in less effort. In the most extreme case where $m = 2$, in order for the principal to deter collusion, the principal ensures the central player gets transfers from her peers in the default equilibrium so she does not initiate collusion. The trade-off between effort and robustness is convoluted. It calls for a more general way to identify the robustness of individuals in arbitrary supervision networks, which I will discuss in the following subsection.

3.4 Arbitrary Exogenous Supervision Network

In most cases, the principal cannot completely control the communication network. Thus, I am also interested in finding the bargaining power of individuals in an arbitrary network. Assume that if the agents are connected in a working relationship (for example, linked as supervisor and supervisee), they must be linked in the communication network. Players can also form additional communication links, with heterogeneous costs and benefits for each link. Thus, I assume the final communication network is in equilibrium. Here, I take it as given and do not model the network formation process. In this case, if one player wants to send a message to an agent she is not directly connected to, she needs to ask some mutual acquaintances to forward her message to the target, as mentioned before. [16] In this model, there is no unmediated connection formation because the original communication is in equilibrium. All possible and profitable communication links should have been formed already before the bargaining stage.

Then, I can check when collusion is possible, given the communication network. The problem of finding the bargaining power in an arbitrary network can be transformed into a linear programming problem. Notice that the effort level for an individual in the default equilibrium affects that individual's bargaining power. The better off this individual is in the default equilibrium, the more she needs to keep to herself in the new equilibrium, which means there is less transfer that she can promise to the other agents. Therefore, in this section, I study two different levels of bargaining power for each individual: the bargaining power given full effort and the bargaining power given maximum robustness without full effort.[17]

The algorithm in this section has significant real-world applications. There are many established methods to elicit personal relationship networks; therefore, the principal can apply the algorithm and calculate the bargaining power of each individual in the department. Then the principal can compare the bargaining power with the technical threshold for collusion and see if the department is at risk of joint collusion. Suppose a player has too large a bargaining power. In that case, the principal has two methods to deal with the problem: First, the principal can use job rotation and replace the central player with a new agent with fewer connections. Then the principal can calculate the bargaining power again and see if the department is still at risk of collusion. If so, the principal can repeat the process until the risk of collusion is minimized. A second method involves reducing the workload of the central player in the default equilibrium and thus reduce her incentive to organize collusion. The third subsection shows an algorithm that can be used to calculate the maximum robustness achieved using this method. Then the principal can decide if a further change to the communication network is necessary.

[16] A concern is that receivers can reject requests for forwarding collusion plans because their forwarded messages would be hard evidence for the collusion plan and may attract punishment. However, it is hard to specify any punishment if the initiator asks only for an agent's phone number. If forming new direct communication cannot be punished, the initiator might use this method to send a direct message to more agents without worrying that the first receiver would refuse to forward the message. However, this strategy cannot be beneficial because it contradicts the assumption that the original communication network is in equilibrium. Suppose the initiator is not connected to a certain agent initially. In that case, the initiator will not be better off establishing this connection in the bargaining stage because all the conditions that block this link are still there. Thus, the initiator has to rely on mutual acquaintances to forward the message.

[17] Both measures of bargaining power coincide in the single ring supervision network, so I do not differentiate between the two in the previous section.

A no-collusion benchmark. I first establish a benchmark of maximum effort in an arbitrary supervision network without considering collusion. The maximum sustainable effort in this case will be used for later analysis. Let S denote the supervision network, r_i denote the number of supervisors of player i, and $r = \sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \sum j \neq i^N s_{ij}$ denote the total number of supervisors in this network, where r equals the number of directed links in the supervision network. In addition, r is also the maximum of net peer transfers that all the players can receive in total. In other words, any equilibrium payoff in a period can be represented as every player i first giving a transfer r_i to a central pool, so there will be r units of utility in the pool. Then the equilibrium specifies a strategy that allocates r as transfers back to the agents or the effort to the principal.

Thus, the principal and the initiator propose an equilibrium to allocate r and maximize their own utility. If there is no negotiation, then the principal can get the full effort that sums to r. In the following subsections, I study the robustness of the network when the principal gets the maximum effort and the effort that the principal can get if he wants to maximize the robustness of the network.

3.4.1 Maximum Effort

The intuition for the algorithm is the following: first, the principal chooses the default equilibrium such that all the agents exert full effort before negotiation. Then, starting from the smallest threshold $m = 1$, I check if each agent can successfully coordinate a joint deviation. If yes, I increase m by 1 and check again if each agent can successfully coordinate a joint deviation. If not, then I say the bargaining power of agent i is $m - 1$. I repeat the process to get the bargaining power of all the agents. The robustness of the network is thus the largest bargaining power among the agents. The larger the number, the less robust the network.

In the remaining of the paper, when describing the algorithm, I describe all the inter-agent transfers as all the agents first putting all the transfers into a central pool, and then the central pool giving the transfers back to the agents. I call the transfer from the central pool to a player i the reward to player i. This description of a central pool is equivalent to the agents directly giving each other transfers, but using the central pool notation, I do not need to specify who gives how much transfer to whom.

Then, I can set up the notation for the allocation of welfare when the collusion plan fails. To optimally deter collusion, the principal will not specify any effort in the equilibrium after collusion bargaining fails. In this way, the principal can specify the highest peer transfer to reward those agents who vote no in the bargaining stage.

Denote the principal's plan of daily reward (peer transfer) to player j as $r_p(j|i)$ if j votes no, where i is the initiator. The principal never gives the "yes" voters any transfer in the new equilibrium ($r_p(k|i) = 0$ for all k who vote yes) because doing so would have no effect on deterring collusion but would increase the probability of the initiator starting the voting stage. The formal proof of this lemma is in Appendix C5. Similarly, let $r_i(j|i)$ denote the initiator's plan of daily reward to player j for voting yes, where i is the initiator.

Then, I define the communication protocol. A communication protocol with respect to a threshold m and an initiator i is denoted by $\sigma_i(m)$. It is an ordered set of $m-1$ receivers who can be reached through communication network $\mathbf{C}$ if all of them vote yes and agree to forward the message. Let $A_i(m)$ denote the set of all such communication protocols when the initiator is i. Let $n(\sigma_i(m))$ be the set of all the neighbors of the players in $\sigma_i(m)$; if the neighbor is in the set $\sigma_i(m)$, then this person is not counted in $n(\sigma_i(m))$.

To simplify the algorithm, I use the following tie-breaking rule.

Lemma 4. *(Tie-breaking rule) Suppose agent i is the initiator. If for all communication protocols $\sigma_i(m) \in A_i(m)$ the initiator needs to give the m receivers a total weekly reward greater than r so that they vote yes, then a collusion proposal of initiator i can never pass.*

When the initiator needs to give strictly more than r units of reward to incentivize m agents to deviate, collusion always fails because the initiator does not have so many resources. The tie-breaking rule solves the problem when the initiator can give exactly r units of reward to m agents. The proof for this tie-breaking rule is in Appendix C4.

More formally, according to the tie-breaking rule 4, the collusion proposal cannot pass if and only if for all communication protocols $\sigma_i(m) \in A_i(m)$

$$\sum_{j \in \sigma_i(m)} r_p(j|i) \geq r. \tag{3}$$

This condition means the principal chooses the equilibrium selection function such that for all communication protocols, the initiator cannot incentivize $m-1$ receivers to vote yes. Thus, the initiator will not choose to start the voting stage. On the other hand, if there is at least one communication protocol that allows the initiator to make $m-1$ receivers to vote yes, then collusion occurs. I call the inequality (3) the blocking condition.

The principal's promised rewards also have to satisfy the feasibility constraint: under each feasible communication protocol of the initiator, the principal's promised rewards to all "no" voters given the communication protocol cannot exceed r. More formally, for each $m' \leq m$, and all $\sigma_i(m') \in A_i(m')$ the following inequality holds:

$$\sum_{j \in n(\sigma_i(m'))} r_p(j|i) \leq r. \tag{4}$$

This condition means the principal's promised transfers to all the agents who reject the collusion proposal must be feasible given any protocols. If this condition is violated, then the resulting outcome will not be an equilibrium.

Lemma 5. *Let $m_i + 1$ denote the smallest voting threshold such that there exists a set of $r_p(j|i)$ that satisfies both the blocking condition (3) and the feasibility condition (4). The bargaining power of the initiator i is equal to m_i.*

With the above definition, I can describe the algorithm of solving for the bargaining power of each individual i in an arbitrary supervision network.

1. Starting with $m = 2$ and an initiator $i \in I$, find if there exists a vector of $r_p(j|i)$ that satisfies both the blocking condition and the feasibility condition.

2. If the solution exists, then this m is the bargaining power of j in this network. Otherwise, increase m by 1 and repeat the first step.

3. Do this for each agent in the supervision network and derive the agents' bargaining power. Then, the maximum of all the bargaining power values is the robustness of this network.

Here, I provide examples of optimal reward allocation in some supervision networks. In the following figure, each arrow points from a supervisor to a supervisee. The letter in each circle is the name of the agent. Suppose A is the initiator, so she is symbolized by a red square. The label next to each node j is the corresponding transfer chosen by the principal: $r_p(j|A)$ represents the case when the agent j votes no and the collusion proposal fails to pass.

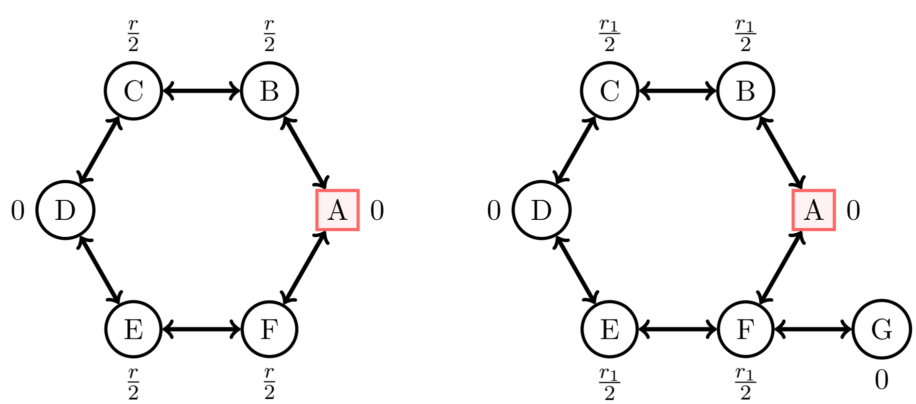

Figure 1: Single-ring supervision network and modification. Agents are represented by letters in circles, and the initiator, by letter A in a red square. Arrows point from supervisors to supervisees. The label next to each node is the corresponding transfer chosen by the principal.

In the figure on the left, the bargaining power of initiator A is $m_A = 2$, which means the initiator A can get at most two yes votes, including her own. Here $r = 12$ is the total transfer in this network. Players B, C, E, and F each can get a $r_p(j|A)$ of $\frac{r}{2}$ for voting no, so the initiator will not have enough reward to incentivize two receivers to vote yes. In the figure on the right, $r' = 14$. The allocation of rewards is the same. However, now $m_A = 4$ because of the additional player G. Though G is not connected to any players other than F, the principal would find no need to give G any reward for rejecting the offer to collude. Intuitively, a player with only one connection has very weak bargaining power. The calculation shows that the bargaining power of player G is $m_G = 1$. It is achieved by the principal choosing $r_P(F|G) = r$.

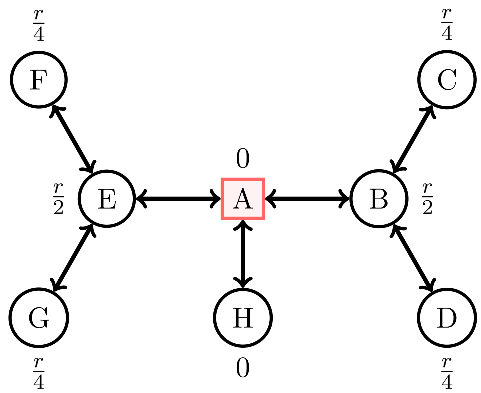

Figure 2: A tree network. Agents are represented by letters in circles, and the initiator, by letter A in a red square. Arrows point from supervisors to supervisees. The label next to each node is the corresponding transfer chosen by the principal.

In figure 2, the bargaining power of initiator A is $m_A = 4$, and the robustness of the entire network is $m = 4$. Being the central player, A has many connections and, naturally, the highest bargaining power. This network is a tree, which means there is only one unique path connecting any two individuals. The feasibility constraint in a tree has a special propriety: Let $r_p(j|i)$ be the principal's reward to player j for voting no. Then let $n_{subtree}(j)$ be the set of players who are directly connected to j in the subtree. Thus, the following should hold:

$$\sum_{k \in n_{subtree}(j)} r_p(k) = r_p(j|i).$$

If the players in the subtree get more than $r_p(j|i)$ in total, such a reward cannot be feasible. If they get less than $r_p(j|i)$ in total, then the principal can give them more reward, making the network more robust yet still feasible.

The above practice can be repeated for all the individuals in a network to get the bargaining power for each of them $m_i(S)$, which means the number of "yes" votes needed to block the deviation proposal from being passed. The following figure lists the bargaining power of each individual in the network.

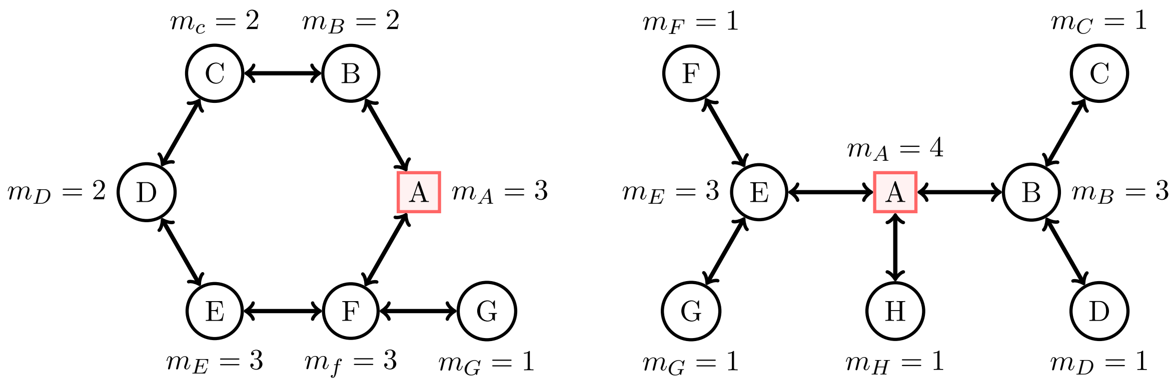

Figure 3: The figure on the left is a modified single-ring supervision network. The figure on the right is a tree supervision network. Agents are represented by letters in circles, and the initiator, by letter A in a red square. Arrows point from supervisors to supervisees. The label next to each node represents the player's bargaining power $m_i(S)$.

3.4.2 Maximum Robustness

In the previous section I used the algorithm to find the bargaining power of each individual when the full effort is required in the default equilibrium. However, the principal can further reduce the agents' bargaining power by reducing central individuals' workload (or even giving them positive transfers) in the default equilibrium. [18] Thus, when the initiator chooses a deviation proposal, she has to give herself more reward so she will not be worse off. Then she will have less reward to incentivize the other agents to vote in favor of collusion. The result is a more robust network, but the principal gets a lower payoff than the one described before. This algorithm has important managerial implications. The intuition is similar to Singapore's "high salary for clean government" policy. Government officials receive a high salary so they have less incentive for corruption. This algorithm thus provides the second policy tool to reduce the threat of collusion. In this section, I describe the algorithm that can be used to find the highest robustness that the network can achieve without requiring full effort.

Let $r_0(i)$ be the transfer an individual i received from the central pool in the default equilibrium. It also equals the total transfers that all the players give away in the default equilibrium. Let $e_0(i)$ be the effort level of individual i in the default equilibrium. The resource constraint requires that

$$\sum_{i \in N} (r_0(i) + e_0(i)) \leq r, \tag{5}$$

where r is the size of the central reward pool if every player gives a daily transfer to

[18] These networks can be more robust than those in which no one exerts any effort; the initiator still has the incentive for collusion and gets transfers from some other agents in the new equilibrium.

the pool that equals the number of the player's supervisors. Call this inequality the anti-corruption reward constraint.

The reward $r_0(i)$ has an anti-corruption purpose because the more reward a player receives, the less motivation she has to initiate a joint deviation. In this section, the principal wants to maximize the robustness without the full-effort constraint, so the algorithm is the following:

1. For each initiator i, starting from $m = 2$, check if there are two vectors $\{r_0(1), r_0(2), ..., r_0(n)\}$ and $\{e_0(1), e_0(2), ..., e_0(n)\}$ such that for each initiator i, there exists a vector of $r_p(j|i)$ so that for each $\sigma_i(m) \in A_i(m)$, the following modified blocking condition holds.

   (6) $$\sum_{j \in \sigma_i(m)} r_p(j|i) \geq \sum_{j \neq i} d_j(S) + e_0(i) - r_0(i)$$

   The right hand side is the disposable reward of the initiator. It is the sum of the total transfers from all the other players plus the effort the initiator can save by deviating from subtracting $r_0(i)$, which is the amount of reward the initiator needs to keep to herself. Moreover, for each $m' \leq m$, and all $\sigma_i(m') \in A_i(m')$, the following inequality feasibility constraint holds:

   (7) $$\sum_{j \in n(\sigma_i(m'))} r_p(j) \leq r.$$

2. If the solution exists, then choose the effort level in the default equilibrium to maximize $\sum_{i \in n} e_0(i)$. The robustness of the initiator i is then $m - 1$.

3. If the solution does not exists, then increase m by 1 and repeat the first step.

Using this algorithm, I can solve for the most robust supervision network. The label next to each node in the following graph is the amount of reward $r_0(i)$ that the corresponding individual gets from the initiator.

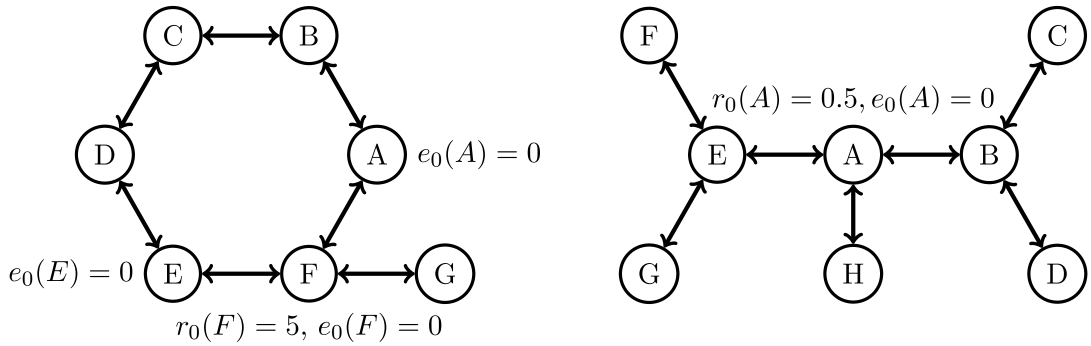

Figure 4: The reward $r_0(i)$ for players in the most robust network. In the network on the left, a player's bargaining power is $\underline{m}(S) = 2$, and in the network on the right, $\underline{m}(S) = 3$.
The figure on the left is a modified single-ring supervision network. The figure on the right is a tree supervision network. Agents are represented by letters in circles. Arrows point from supervisors to supervisees.

Figure 4 indicates the $r_0(i)$ and $e_0(i)$ of each agent to maximize the robustness of the network. Players without a number on the side exert full effort and receive no transfer in the default equilibrium. Compared to the full-effort cases, the left network's robustness goes from 3 to 2, and the right network's robustness goes from 4 to 3.

With the following exercise I aim to improve the robustness of the network. In the left graph of Figure 4, players A, E, and F do not need to exert effort in the default equilibrium. In addition, player F gets 0.5 units of total transfer from players B, C, D, and G. Thus, in the default equilibrium, the total effort from B, C, D, and G is reduced from 7 to 6.5, which is the amount that the principal gets.

In the graph on the right in Figure 4, player A exerts zero effort in the default equilibrium. In addition, all the other players give A a total transfer of 0.5 units per period. Thus, the total effort that the principal can get during the default equilibrium is 10.5 units per period, which is much less compared to 14 in the full-effort case. Finally, the principal can check if reducing the effort level in the default equilibrium can make the equilibrium more robust and thus avoid collusion.

To complete the story, I show that limiting the agents' communication effectively reduces their bargaining power. The formal proposition is the following:

Proposition 5. *For a given set of agents* $\mathbf{N}$*, there is a communication network* $\mathbf{C}$*, and there is a supervision network* $\mathbf{S}$*. In both the maximum-effort and maximum-robustness settings, cutting any communication link in* $\mathbf{C}$ *(and the corresponding supervision link in* $\mathbf{S}$ *if necessary) weakly decreases the bargaining power of all the agents.*

This proposition is one of the main points of this paper: reducing communication helps to deter collusion. The proof is the following: if only a communication link in $\mathbf{C}$ is cut while the supervision network remains the same, then the set of communication protocols for each initiator to send a message to $m-1$ agents (for all $m \geq 2$) decreases weakly. Thus, the bargaining power of the initiator cannot increase, because the principal can use the same equilibrium selection rules to guarantee the same bargaining power of each agent after cutting the communication. On the other hand, with fewer communication protocols, it might be possible to stop collusion with a smaller m, so the bargaining power of each agent may decrease.

If both the communication and a corresponding supervision link are cut, then the relocatable resources for both the initiator and the principal decrease by 1 unit. Thus, they still have the same amount of resources to incentivize or deter collusion, so the bargaining power of each agent depends only on the communication network. Since there are fewer links in communication network $\mathbf{C}$, the set of communication protocols for each initiator to reach the other $m-1$ agents also shrinks weakly. Thus, the bargaining power of each agent also decreases weakly. This completes the proof.

3.4.3 Comments

The two algorithms in this section can be used in models with more realistic assumptions. For instance, one concern is that forwarding messages can be costly for agents. It is possible to incorporate this communication cost by subtracting it from the initiator's relocatable resource r. Then, I check if the initiator can still pass the deviation with the reduced resources. It is also possible to analyze agents with different preferences such as altruism, concern for fairness, and conscience,[19] and heterogeneous losses from effort or punishment. These additional elements also affect the relocatable resources of the principal and initiator. Nevertheless, I can still apply the two algorithms in this section and calculate the individual bargaining power.

Finally, the two algorithms in this section imply that external reward or punishment is ineffective for deterring collusion if the colluding agents can fully manipulate all contractile signals. In practice, the principal has to specify a clear set of conditions to determine who should be rewarded and who should be punished. However, when a group of agents colludes, they can fake evidence in their favor and get the reward, while the non-colluding agents are incorrectly punished. For instance, if the punishment and reward are based only on the agents' testimony, the colluding party can coordinate their words and make false claims to exploit the system.

As a result, contrary to intuition, the external reward increases the relocatable resources of the initiator because she can share the external rewards among the colluding agents and

[19] The people who have conscience inherently prefer not to collude, so they suffer additional utility losses when collusion occurs.

increase their incentive to collude. On the other hand, external punishment makes it more costly for agents to reject collusion, because those agents will be falsely accused and suffer additional punishment. Consequently, if the external reward and punishment are subject to manipulation, they cannot help deter collusion. The highest robustness is always achieved in a fully autonomous department. External punishment and rewards will only be effective if they are based on hard evidence that the colluding party cannot fake. Thus, a principal needs to thoroughly assess if all the evidence used is credible and immune to manipulation. If not, then a fully autonomous department might be the best choice to deter collusion.

4 Extension and Discussion

4.1 Managerial Implications

The results in this paper have several practical managerial implications. When the colluding party can manipulate evidence, external punishment and reward make collusion more likely to happen. Thus, to deter collusion optimally, the principal needs to rely on peer supervision.

To check if a department is at high risk of collusion, the principal first needs to calculate the threshold for successful collusion. Doing so involves conducting a detailed inspection of the internal control procedure and finding the loopholes that facilitate collusion. Once the threshold for successful collusion is known, it is possible to establish a communication network among the agents using established empirical methods. Then, using the algorithms in this paper, the principal can check if there are agents who can successfully organize collusion.

Suppose the result shows that the department is at high risk of collusion. In that case, the principal can do the following things: (1) modify the communication network by job rotation to cut excessive informal communication links, or reform the working relationships to make communication links sparser; (2) reduce the working load of the central players or increase their compensation; (3) change the internal control practice and make collusion harder to organize (i.e., increase the success threshold for the collusion proposal to pass).

4.2 The Advantage of Peer Supervision

When the principal cannot use punishment or reward to deter collusion, conventional mechanism design suggests the principal "rent out" the department and let the agents achieve the highest payoff using their power. The principal then charges a fixed amount of rent and makes the agents indifferent between working in the department and their outside choices. However, I recognize three main advantages of the peer supervision mechanism compared to the conventional solution: risk sharing, liquidity constraint, and externality control.

First, if the agents are more risk averse than the principal is, renting the department to the agents and letting them bear all the risk of aggregate output shock may cause welfare losses. The peer supervision structure allows the agents to maintain a constant effort level while the risk of aggregate output shock is on the principal.

Second, if the agents are liquidity constrained and cannot afford to rent the department, the result is welfare losses.

Externality control is a more important point. When the agents maximize their benefits without supervision, such activity can generate a large negative externality. For instance, privatizing the police department can be a terrible idea. Suppose the police can fake evidence and the officers cover one another for their criminal activities, the principal (i.e., the general public) cannot use punishment or reward based on the evidence raised by the police department. Thus, the police abusing their power would cause large social welfare losses.

Here, I provide a simple mathematical explanation for externality. Let u_i denote the utility of the outside career option of each police officer. Let π denote the total profit of the department if the police officers can freely abuse their power. Suppose the government charges a fixed rent r for the private company that runs the police department such that $\sum_i u_i = \pi - r$; thus, the police officers are willing to work. Then, one can anticipate the private police department to abuse their power and conduct illegal activity to achieve the maximum profit π. However, that illegal activity may generate a large negative externality: for each dollar of profit the police officer acquires, society suffers a loss much higher than one dollar. A much better option is to let the police officers exert effort, supervise each other, and not to abuse their power. Letting e_i be the individual effort level results in $\sum_i u_i = \pi - \sum_i e_i$. Each police officer is still willing to work, and a negative externality is not generated. The total social welfare can thus be much higher under the peer-supervision scheme than in the no-supervision case. The government can also compensate the police officers with a fixed amount, so a higher effort level is sustainable. The compensation to the police is collected through less harmful channels such as taxes or other state-owned businesses. This example shows the importance of having a proper peer supervision structure.

4.3 Real-world Applications and Examples

There are many examples of how limited communication helps the principal deter collusion. One example is the 2016 Turkish coup attempt. The initiators of the coup needed to get a certain amount of military personnel to support them to succeed. During the initial stage, some of the top Turkish military officials were taken as hostages by the coupists. They forced the generals to sign the coup declaration, which was a way to send the message to the military sector that each general was in charge of. Assume the coupists did not have the channel to contact lower-rank military staff directly. However, some generals refused to sign the declaration (such as Turkish Chief of the General Staff Hulusi Akar) and publicly ordered all personnel to return to their barracks. Thus, the total support for the coupists decreased significantly.

The coup allegedly failed also because the conspirators were unable to seize control of the media and frame the narrative, according to Naunihal Singh, author of the book *Seizing Power*. The mass media must be under the rebels' control for coups to succeed, but in 2016, the Turkish mass media was still under the control of President Recep Tayyip Erdoğan. So, it was even harder for the coupists to gather support, and eventually, the coup failed because of lack of support (i.e., the collusion proposal did not pass the success threshold).

When Baron de Montesquieu (De Montesquieu, 1989) first introduced the idea of the separation of powers (the legislative, executive, and judicial powers), he envisioned the balance of power would create a higher-order structure that maximizes social welfare. If one of the powers deviates, the other two should identify and correct the mistake. The assumption is that the three powers cannot form a grand coalition. If a collusion forms, the three powers can jointly exploit the general public to maximize their private welfare. In such an eventuality, ordinary democratic practices such as elections can no longer be effective because it would be easy to fake election results. The assumption of no overall collusion is generally true because grand collusion of the three powers may leave a large amount of traceable evidence. If one branch refuses to join, the people can be informed about the collusion attempt and force a restart of the entire government, so the initiator would not be able to incentivize collusion (i.e., have enough rewards to convince the others to collude). However, one cannot rule out the possibility that some minor government branches can collude; this situation is usually referred to as corruption. For such minor corruption scenarios, this paper provides both a measure of the collusion risk and policy tools for deterring collusion.

The separation of duties is also critical for firm management. It is easy to deter collusion if one agent can reject it and credibly report the collusion attempt. The principal can reward the first agent who rejects the collusion proposal with $r/(n-1)$ units of payoff for reporting the incident, where r is the total welfare gain from collusion, and punish the colluding agents. This strategy is commonly used by firms and is usually adequate. However, in some other cases, detecting a collusion attempt is not always so simple. For instance, multiple agents can jointly fake evidence and accuse the other one of being the actual initiator and deserving punishment. When this situation occurs, the higher-order managers usually believe the story of the majority and dismiss the evidence provided by the agent who in reality rejected the collusion proposal. The problem may be more severe when some agents have greater authority over others, and the threshold for joint deviation may be well below half of the department size. If a firm is aware of such a threat, rewarding the whistleblowers is not enough to deter collusion. As predicted by the theory model, if the collusion threshold is small, a constraint on communication will also be necessary to deter collusion.

The managerial implications of these findings are the following: First, make it clear that any collusion attempt is punishable even if the agents have not engaged in misconduct yet. Second, periodically "restart the department" using job rotation, to break the undetected collusion attempts. Doing so limits the amount of excessive personal connection among the workers and thus increases the robustness of the collusion deterrence methods. Third, it is essential to construct ways for a small fraction of agents to reliably report misconduct (making the success threshold m larger) so making up fake evidence becomes harder.

4.4 Collusion by Commitment

In Appendix D, I discuss a different way of forming joint deviation: commitment. Assume the initiator can commit to any strategy and invite the other agents to join the commitment group. The other agents can then choose whether to join the group. It is a much stronger way of collusion formation. I will show that the joint deviation to no effort cannot be stopped with unlimited communication. However, limiting the communication network can still effectively deter collusion.

5 Conclusion

This paper models the detailed negotiation process of equilibrium selection. The model suggests that the players cannot always coordinate to a Pareto optimal outcome and thus may be stuck in an inefficient equilibrium: the sparser the communication network, the more complex it is for the players to coordinate a joint deviation. Thus, a principal can take advantage of this finding and make a group of agents exert stable effort without direct supervision. In general, the principal can make the agents supervise each other and limit the communication among them to deter collusion. This peer supervision model has many real-world applications in firm management, political systems, and more.

This endogenous equilibrium selection model can also be applied to more general settings. First, it is necessary to specify the communication network corresponding to the stage game and the criteria for joint deviation. Then, one can model each equilibrium's stability and what equilibrium the players will choose through the endogenous equilibrium selection process.

An interesting research avenue would be studying other endogenous equilibrium selection processes, such as subgroup coalition. However, the strategy space for such a coalition is so ample that I leave it for future research.

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6 Appendix

A1. An example effort provision equilibrium of lemma 1

Here, I provide an example of effort provision equilibrium. The players may use minmax punishment to threat those who does not exert enough effort.

At the first stage (morning) of each period, for every player $i \in \mathcal{I}$, choose effort level $e_{it} = n - 1$ and zero transfer $\pi_{ijt} = 0$ for all $j \in \mathcal{I}$ and $j \neq i$. If no deviation happens at the morning, choose $\phi_{i,j} = 0$ for all $iandj \in \mathcal{I}$, in other word, no punishment if no deviation. The game repeats for all future periods. If a player i deviates at the morning of a period t, then the game changes to a punishment stage. In the second stage (afternoon) of the day t, all the other players $j \neq i$ choose $\phi_{jit} = 1$ to punish the deviater. If everyone carries out the punishment accordingly, then the game restarts on the next morning, and everyone is expected to exert effort $e_{i(t+1)} = n - 1$.

If a player $k \in \mathcal{I}$ deviates in the punishment state, then all the other players switch to punishing this player k for all later periods by choosing $\phi_{j,k} = -1$ for all $j \neq k$. Deviation in the punishment state will result in a payoff stream of $0, -n+1, -n+1, ...,$ for the deviater, while not deviating will result in $-1, -1, -1, ...$ There is no profitable deviation in the punishment state if $\frac{\delta(n-1)}{1-\delta} \geq 1$. The inequality simplifies to $\delta > \frac{1}{n}$. Thus, if $n > 2$, the equilibrium holds when $\delta \geq \frac{1}{2}$. If there is a new deviater in the punishment stage, then all the players (including the original deviater) switch to punish the new deviater. Deviation in the punishment stage results in more severe long term punishment because the other players can only respond to such deviation in the following day, and thus the loss to the deviater is discounted.

Finally, ignore multiple simultaneous deviations. Because we are looking at the subgame perfect Nash equilibrium, so long as the strategy can deter all unilateral deviations such strategy should be a SPNE.

In this paper, we mainly focus on patient players, which means $\delta \geq 1/2$. However, the main results of this paper also holds when the patience level is even smaller. The main problem with impatient agent is that they might lack the incentive to carry out punishment. To deal with this issue, the principal can reduce the required effort level of each agents accordingly. So less number of peers are needed to punish those who fall short of effort. Consequently, those who carry out punishment can be rewarded by transfers from those who are not required to exert punishment on the previous day. If someone deviates from the punishment path, she not only lose the reward, she would also be punished by the others on the next day. Using such an equilibrium, positive effort level can be sustained with very small δ as long as n is sufficiently large. □

A2. Proof of corollary 1

Here, the principal is not a player in the game. Though reducing the effort would decrease the principal's utility, since she cannot exert punishment or reward on the agents after the game starts, she is no longer an effective player. Thus, I assume the agents can renegotiate without the principal's involvement.

First, I show that any effort provision equilibrium is not coalition proof. According to Bernheim et al. (1987), for an equilibrium to be coalition proof, it must be efficient within the category of self-enforcing agreements, where self-enforceability requires that no coalition can benefit by deviating in a self-enforcing way.

Suppose there is an efficient effort provision equilibrium within the category of self-enforcing agreements and thus coalition proof. Consider an alternative equilibrium in which all the players play the same strategy. However, whenever the original effort provision equilibrium requires a player to exert a positive effort level e, the player is no longer required to provide this effort in the new equilibrium. Since the original equilibrium is self-enforcing, the alternative equilibrium must also be self-enforcing because the interactions among the players are not changed. Then, the original effort provision equilibrium cannot be efficient within the class because the alternative corrupt equilibrium is a Pareto improvement from the original one. Thus, the effort provision equilibrium cannot be a coalition-proof equilibrium.

Next, renegotiation proof (Farrell and Maskin (1989)) requires that before the beginning of each stage, the continuation payoff of the equilibrium is not Pareto Dominated by any other equilibria. Obviously, the effort provision equilibrium fails this condition because it is dominated by a corrupt equilibrium, as shown in Proposition 1. □

B1. Proof of Lemma 2

The proof is the following: if a deviation to a new equilibrium eq can be passed, yet some players still need to exert a positive amount of effort, then eq cannot be an optimal deviation proposal of the initiator. The initiator can be strictly better off choosing another equilibrium eq' with the same punishment and reward structure, but whenever the effort level is $e_j > 0$ for each player j in eq, the alternative equilibrium eq' has strategy $e'_j = 0$ and $\pi'_{ji} = e_j$ for all agents whose $e_j > 0$ and i is the initiator. This change from eq to eq' means, the initiator collects all the benefit of effort and keep them to herself. Though this change, all the other players has the same utility level in all the sub-games, but the initiator is strictly better off.

B2. Proof of proposition 2

To prove this proposition, we should start with small voting threshold m.

If m=1, then the initiator only needs her own vote to pass the deviation plan. So she would select the equilibrium such that everyone else give her a transfer of $n-1$ everyday while no one exert any effort. As showed before, such an subgame perfect equilibrium exists. In the new equilibrium, the initiator gets a stage game utility of $(n-1)^2$. To stop such a deviation, the principal needs to promise each agent at least $(n-1)^2$ utility in each period, however, that is not possible without external source of transfer. Thus, when $m=1$, deviation cannot be stopped for all $n \geq 1$.

If m=2 and $n \geq 2$. Then, we can use backward induction to solve the voting stage. To stop the deviation, the principal needs to make every receiver vote for no. The principal thus choose an equilibrium so that each of the receivers who votes for no gets a per-period utility level $u_i = r_2$. The 2 in the subscript of r_2 denotes the reward to the receiver when there were $2-1$ yes voters (the initiator in this case) prior to the receiver. The reward to each of the receiver for rejecting the offer must be the same. The reason is that the initiator only needs to induce one receiver to vote for yes then the deviation will be passed, so if the reward to each receiver is different, the initiator would only need to bribe the receiver who gets the least reward from the principal. And that is going to make all the excess reward to the other agents useless. So all the rewards for rejecting the deviation offer must be the same. Thus for all the receivers to reject the offer, the total rewards required is $r_2 * (n-1)$.

When the deviation offer is rejected, the reward that amounts to $r_2 * (n-1)$ has to come from the initiator, because all the other agents' per-period payoff has to be r_2. The maximum reward that the initiator can give out is $n-1$ if the votes fail (proved in lemma 1). Thus, $r_2 * (n-1) \leq n-1$. So $r_2 \leq 1$ defines the upper bound of the reward. So, the principal can choose $r_2 = 1$ to maximize the deterrence for deviation.

On the other hand, the initiator only needs one receiver to vote for yes. In the repeated game, the initiator is willing to give at max e_0 transfer to the yes voter every day, so she is weakly better off than not initiate the voting stage. If one receiver votes for yes, then each of the remaining (n-2) agents can give out (n-1) unit of transfer everyday. Thus, the total transfer to the yes voter is $e_0 + (n-1)(n-2)$.

So the deviation will be passed if the reward to the one yes voter is weakly larger than the promised reward from the principal [20]. So the deviation will be passed if $e_0 + (n-1)(n-2) \geq r_2 = \frac{n-1}{n-1} = 1$. This inequality is true for all $e_0 \geq 0$ when $n \geq 3$. And it cannot be true if $n = 2$ and $e_0 < (n-1)$. In other words, we can conclude that the deviation to corruption can be stopped only when $m = n = 2$ by setting an default effort level strictly smaller than 1. If $n \geq 3$ and $m = 2$, the deviation always happens even in the default equilibrium the

[20] The initiator can always promise $r_2 + \epsilon$ rewards to the receiver for passing the vote. Then the receiver always strictly prefers to vote yes for the deviation plan. Since $\epsilon > 0$ can be arbitrarily close to zero, we can simplify the analysis by assuming the tie breaking rule: if the initiator promise the same utility to the receiver as the principal does, the receiver always vote for yes if the vote can be passed

effort level is zero.

When the deviation can be passed, the initiator would choose a case that everyone except the initiator and the first yes voter to make an outward transfer of $n-1$ everyday. So in total, the transfer is $(n-1)(n-2)$ The first yes voter gets a fraction equals to $r_2 = 1$, and the initiator gets $(n-1)(n-2)-1$.

When $m \geq 3$. We can still use backward induction. Consider the vote of the receiver when there were $m-1$ yes votes before her (including the yes vote of the initiator). The maximum reward that the principal can promise to the rejecter i at that point is $u_i = r_m = \frac{(n-1)(m-1)}{n-(m-1)}$. The numerator comes from the following reason: If every receiver rejects the offer, they can exploit the the first (m-1) yes voters and make each of them giving out a transfer of (n-1) everyday. So the total available transfer is $(n-1)(m-1)$. The $n-(m-1)$ in the denominator is the number of rejecters who share the total transfers. So, for the deviation vote to pass, the initiator needs to promise the last receiver a transfer of at least r_m. (All the no voters and those who are called after the mth yes voters pay this reward.)

Let x denotes the stage of the voting game. When the first receiver gets the call from the initiator, it is stage 1. For each receiver that votes for yes, add one to the stage count. The principal can also promise reward r_{m-1} and try to make all the $m-1$ stage receivers vote for no. We can calculate the reward:

$$r_{m-1} = \frac{(n-1)(m-1)}{n - \mathit{numYes}_{m-1} - \mathit{numIneffective}_{m-1}}$$

The numerator $(n-1)(m-1)$ is the total transfer giving out by the first $m-2$ yes voters and the last ineffective voter together. The denominator is the number of remaining effective voters who share the total transfers. $\mathit{numYes}_x = x-1$ is the number of yes votes at voting stage x. The $\mathit{numIneffective}_x = m-(x-1)-1$ stands for the number of ineffective voters when there are $x-1$ yes voters. It means, if all the ineffective voters votes yes, there will only be $m-1$ yes votes in total by the end of the voting stage. So the deviation plan cannot be passed. For instance, if there are only m-2 yes voters, all the others voted for no except the last person contacted, in such a case, the no matter what the last agent votes, the deviation will not be passed, so the last player is called the ineffective voter.

Reorganizing the terms and we have, $r_{m-1} = \frac{(n-1)(m-1)}{n-(m-1)} = r_m$ By similar reason, $r_x = \frac{(n-1)(m-1)}{n-(m-1)} = r$ for all $x \in \{2, 3, 4, ..., m\}$ Thus, the initiator has to promise a reward of r to each of the receiver so that the deviation vote can possibility be passed. All the rewards come from the no voters and ineffective voters (n-m people) [21] Initiator's total disposable reward is $e_0 + (n-1)(n-m)$. So the deviation will be passed if the following inequality holds:

[21] unreached voters are those who have not received a phone call when m yes votes are gathered.

$$(n-1)(m-1)^2/(n-m+1) \leq e_0 + (n-m)(n-1)$$

Rearrange the term:

$$m \leq \frac{n^2+n-1+\frac{(n+1)e_0}{n-1}}{2n-1+\frac{e_0}{n-1}} = \frac{(n-1)(n^2+n-1)+(n+1)e_0}{(2n-1)(n-1)+e_0} \tag{8}$$

Since every period, an initiator occur with probability p_0. As time goes to infinity, an initiator appear almost surely. If m is small, then the principal cannot provide sufficient rewards for any receiver to vote no, and the deviation will always be passed. In this case, voting no is a weakly dominated strategy, so by the refinement of no weakly dominated strategy all $m-1$ receivers vote for yes. If m is large, then the principal has enough reward so there will be at least one step that all the receivers vote for no. So that the deviation cannot be passed under any possible equilibria. Thus the proposition for sequential voting is proved.

The right hand side of the bound is increasing with e_0. In other word, the more effort in the default equilibrium, the more likely the agents collude. The tightest bound for m is thus achieved by $e_0 = 0$, and this gives the bound in the proposition. Notice that when $e_0 = 0$ the bound is strictly smaller than half of the department size for all $n > 0.5$. It means, if

For larger m, the principal can specify a higher effort level. For m sufficiently large, the principal can specify full effort $e_0 = n-1$.

We can plug in $e_0 = n-1$, the bound on m becomes the follow:

$$m \leq \frac{n^2+2n}{2n} = \frac{n}{2}+1 \tag{9}$$

Thus, if m is larger than half of department size plus one, then full effort can be supported.

Then, we need to show that the principal cannot do better. Proof by contradiction: suppose that m satisfies the condition 8. Then, the initiator would always be able to get at least m yes votes, because not it has enough resource to always pay a little bit more than what the principal promises.

Finally, I show that if on the equilibrium path the agents cannot coordinate a joint deviation, they also cannot coordinate a joint deviation on the punishment stage.

1. Since the voting stage happens before the effort and transfers takes place, if someone deviates in the effort and transfer stage, it is not possible to coordinate a joint deviation before the deviater gets the punishment on that day. On the next day, there will be no further punishment so we do not need to worry about that.

2. If someone deviates on the punishment stage, then this deviater will be punished on the next period. So, if there is a voting stage in the beginning of the next period,

then the incentive problem for the players will be different. We first look at the full effort case. Suppose the initiator is not the deviater on the previous punishment stage: When there are at least three agents in the game, the continuation payoff for the initiator to stay at the punishment stage is strictly smaller than on the equilibrium path. Because on the punishment stage, she only needs to punish one deviater and no effort or transfer is required, while on the equilibrium path the stage game payoff is $n-1$, which is strictly worse. So if the initiator does not have the incentive to coordinate a deviation on the equilibrium path, she also does not have the incentive to do so on the punishment path. If there are only two agents in the game, then this arguments depends on the tie-breaking condition. But still, the initiator does not initiate the vote.

If the initiator is the previous deviater, then her continuation payoff on the punishment path will be the same as the continuation payoff on the equilibrium path. So if on the equilibrium path the initiator cannot coordinate a joint deviation, she also cannot do that on the punishment path.

Finally, if full effort is not required in the equilibrium. The principal can still deter collusion on the punishment path by not "over punishing" the deviater. Suppose player i deviates from the punishment stage on period t. Let e_0 be the effort level in the equilibrium path. So, the magnitude of the punishment is $p_{ijt} = \frac{e_0}{n-1}$. To stop such a deviation, on the following day, the other agents should give a total punishment that equals to $\frac{e_0}{\delta}$ [22], which means each agents j gives the deviater i a punishment of magnitude $\frac{e_0}{\delta(n-1)}$. We look at the case when $n \geq 3$. The stage game continuation payoff is not worse for any player in the punishment state. So no one has more incentive to coordinate a deviation.

Thus the proof is completed. □

C1. Proof of proposition 3

Since the communication network is a ring, if there are two receivers votes for no, then the initiator will not be able to contact any other player, because she would have exhausted her list of phone numbers by then. Since the proposition requires $m \geq 3$, the initiators needs to make at least two receivers vote for yes, so that the joint deviation is possible. However, I will show that the principal can always make it impossible to have enough yes voters.

Then, the principal can deter all deviation proposal by choosing the following equilibrium selection function eq_p: First, the effort level in the default equilibrium is e_0 which is strictly smaller than 2, but can be arbitrary close to 2. So full effort is supported. The eq_p specifies that for the first two receivers who votes for no (the gate keepers), each gets a reward of

[22] suppose n is large and the agents are relatively patient so $\frac{e_0}{n-1} \leq \frac{e_0}{\delta}$. So this way, the future punishment is can guarantee the punishment today is incentive compatible, and the deviater on the punishment path is not overly punished to have additional incentive to coordinate a joint deviation.

$(n-2)(2)/2$ every day. Such rewards comes from the players who voted for yes and those who are unable to vote. Each of the others give the two no voters a transfer equal to 2 units of utility every day. No one is required to exert any effort afterward. So, everyone (except the no voters) gets a stage game payoff equals to -2. The -2 is the new minmax payoff since everyone only has two supervisors now. (I will show later that such an subgame perfect equilibrium exists.)

Thus, the initiator can promise the two gate keepers a total reward of $(n-3)(2)+e_0$ for them to agree to the deviation. Since the initiator cannot expect to be worse off after initiating the votes, the initiator her self can give a total transfer to the yes voters no more than e_0. All the rewards have to come from the rest $n-3$ individuals.

If the initiator promises the first receiver a daily reward no less than $(n-2)(1)$, then the first receiver would accept. However, the initiator would have less than $(n-3)(2)+e_0-(n-2)(2)/2 = n-4+e_0$ rewards left, which is strictly smaller than $(n-2)$. So the next two receivers would vote for no and the deviation will fail.

If the initiator promises the first receiver -2 units of daily payoff, then the first receiver would always rejects the offer. So, if the deviation is rejected, then the principal needs to give the first receiver a daily reward of $(n-2)$ as specified in the principal's equilibrium selection function. Then, for the vote to pass, the initiator needs to get at least another two yes votes before the next rejection. So the initiator needs to promise the second receiver a reward of at least $(n-2)$. Thus, the remaining reward for the initiator is now $n-4+e_0$ which is strictly smaller than $(n-2)$. So the third receiver would vote for no.

We can see that if the deviation is welfare improving for the initiator, she can get at most one receiver to vote for yes. So, any deviation plan that can be passed when $m \geq 3$, the initiator must be worst off. As a result, the initiator does not have the incentive to start the deviation vote.

A special case is when $m=3$ and $n=3$. The principal promises each receiver a reward of $(n-2)=1$. If the initiator wants both players to vote for yes, then the total rewards that she needs to give out is 2. Since $2 > e_0$, so propose the deviation makes the initiator strictly worse off. Thus, the deviation does not occur.

C2: Proof of Corollaries 2 and 3

First, I show that when $m=2$ and $n \geq 3$, the deviation to corruption will always be passed. Because if there are at least three players, then at least one of them will have two direct links. Call this player with at least two links as player A. If A is the initiator and $m=2$, she only needs one receiver to pass the deviation plan. So, the principal has to make both receivers reject the offer. However, the largest reward for rejecting the offer is at most $r/2$ for each receiver. So if the initiator exerts full effort in the default equilibrium, she is always willing to give $r/2 + \epsilon$ to the first yes voter. Thus, the deviation must occur when an agent has two neighbors and $m = 2$. We assume all the players must be connected in each supervision network, there must be an agent with at least two neighbors. Thus when $n \geq 3$ and full effort is required, the most robust network S has $m(S) = 2$. Since a ring has the robustness of 2, it is weakly more robust than all other supervision networks.

Reducing the supervision link must reduce the effort level but cannot make the network more robust. A linear supervision network has the same robustness as a ring. However, the sum of effort is smaller in a linear network than in a ring because the two agents on the endpoint have fewer supervisors. So the principal would strictly prefer a ring to a linear supervision network when $m = 3$.

The principal must have more supervision links than a ring to generate more effort than a single ring. However, more links strictly reduce the robustness of the network. When $n = 3$, the proof is trivial because you cannot have more edges than the single ring supervision network. For any network with $n \geq 4$ agents, if there are strictly more than n links, there be an agent with at least three neighbors. If this player is chosen to be the initiator, she has a bargaining power of 3 which makes the entire network strictly less robust than the ring. Thus, both corollaries are proved.

C3: The Star Network

As mentioned before, the star network is the most robust. In this subsection, I briefly characterize the corresponding effort and transfers. Let player 0 be the central agent, and let player $\{1, 2, ..., n\}$ be the peripheral players. $S_{0j} = S_{j0} = 1$ for all $j \neq 0$. $S_{ij} = 0$ otherwise.

In this supervision network, the total disposable reward is $2n$ everyday. Let $r_0(0) = 2n - 2$ and $r_0(j) = 0$ for all $j \in \{1, 2, ..., n\}$. Whenever a peripheral player is chosen to be the initiator, she can only call the central player. The principal can promise the central player a reward of $2n$ for saying no, then the peripheral will never be able to start a deviation.

On the other hand, if the central player is the initiator, she has to give herself a reward of at least $r_0(0) = 2n - 2$. Consequently, for each receiver, she can only promise a reward smaller than 2. However, the principal can choose $r_1(j|centralplayer) = 2$, and all the peripheral players would reject the deviation offer. Thus, all the players have $m_i(S) = 2$. However, if the equilibrium is constructed in this way, the principal can only have a total daily effort of 2 no matter how many agents are there in the department. So, increasing the robustness from the single ring to the star network is extremely costly.

C4: Arbitrary network tie-breaking rule.

The tie-breaking rule solves the problem when the initiator can give exactly r units of reward to m people. Here, I want to show the reasoning behind the tie-breaking rule, and why it is reasonable to assume the collusion fails in this knife edge case.

In this case, I argue the principal can still deter collusion because the principal can reduce the effort level in the default equilibrium for all the agents by a small amount $\epsilon > 0$. So, for the initiator to not be worse off after the collusion is successful, the initiator needs to keep a reward equal to ϵ to herself. So the total reward she can allocate to the other agents reduces to $r - \epsilon$. Thus, there will be at least one of the m players in each communication protocol strictly better off voting no instead. So, the deviation will not be passed. Anticipating that the collusion will fail, all the receivers will vote for no, and the initiator will not even start the bargaining process.

C5: No reward to the opposite side

In this section, I show the following lemma:

Lemma 6. *The principal will not specify any reward (peer transfer) to yes voters. Similarly, the initiators will not specify any reward (peer transfer) to the no voters.*

We can start by analyzing the principal's decision. When a receiver decides whether to vote yes or no, she only compares the rewards she will get in the following four cases. 1. she votes yes, and the collusion is passed. 2. She votes no, and the collusion is passed. 3. she votes yes, and the collusion fails. and 4. She votes no, and the collusion fails.

When the payoff in case 3 is less than in case 4, if the principal rewards some of the yes voters when the collusion fails, then such a policy does not affect the decision of the yes voters. However, the principal has less reward to incentivize the other voters to choose no. So, there will be weakly fewer agents who vote for no in total, and the network will be less robust. When the payoff in case 3 is higher than in case 4. Then, the principal can make weakly fewer people vote yes by simply reducing the rewards to yes voters so the payoff in case 3 is lower than in case 4. So combining the discussion above, rewarding the yes voters when the collusion fails cannot be an optimal strategy to maximize the robustness.

Similarly, the initiator will not allocate rewards to the no voters. We can see this by comparing welfare obtained in case 1 and case 2 above.

D1: Deviation by Commitment

This section studies a stronger version of coalition formation: commitment. I show that if players can commit, they always commit to a Pareto Efficient strategy profile in a fully connected supervision network. However, such a deviation can still be stopped under a single ring supervision network.

The timing of this model is similar to the previous models. The main difference is that a commitment stage replaces the voting stage. Assume that in every period, an initiator appears with probability p. Once an initiator chooses to start the commitment stage, there will be no future initiator (Because the commitment cannot be changed). Assume that the initiator has a commitment device. Using this device, a player can commit to any strategy in any history regardless of whether such strategy is incentive-compatible or not. The initiator first chooses a commitment selection function and then calls each agent to ask whether they want to join the commitment plan (similar to the sequential voting stage). The commitment stage ends if all the receivers are reached once or if the initiator chooses to end the stage.

More precisely, the initiator proposes a commitment selection function: $C_i : V \to S$, where $S = \{s_1, s_2, ..., s_n\}$ is the set of strategy for each player. $V = \{v_1, v_2, ...v_n\}$ is the set of commitment from the agents. For receiver i, $v_i \in \{yes, no, NA\}$, in which "yes" means that the receiver i agrees to join the commitment proposed by the initiator. "no" means the receiver i will not follow the commitment function but follow the equilibrium selection function of the principal instead. "NA" means the agent is cannot be reached by the initiator and thus she also follow the principal's equilibrium selection function. Denote the set of agents who vote for yes as G_{yes}, and I shall refer to it as the commitment group and denote the set of all the other agents as a no-commitment group G_{no}. S is the set of strategies for all the players. Let S_{yes} denotes the set of strategies for those in G_{yes}, Let S_{no} denotes the set of strategy for those in G_{no}. Since the commitment only applies to those who agree to join, the initiator can only choose S_{yes} (the commitment strategy), while S_{no} is determined by the equilibrium selection function of the principal.

The equilibrium selection function of the principal is similar to the one in the previous section. The difference is that the equilibrium selection function here depends on the initiator's committed strategy. Denote the equilibrium selection function of the principal as $eq_p : V \times C_i \to S$. This function only selects subgame perfect strategy for players in G_{no}.

Since the initiator is one of the agents, she can also observe the equilibrium selection function of the principal and best respond to it. Again, to deal with the potential multiple-equilibria problem, we also assume that all the players would not choose a weakly dominated strategy.

When the bargaining stage starts, assume that the history of the negotiation stage is only observable to those who receive the phone call from the initiator. All the other agents observe the history of the negotiation when it ends.

To make this model non-trivial, I make the following assumption:

Assumption 3. *The initiator cannot commit to give transfers to players who are in g_{no}.*

Without assumption 3, the initiator can make any player do anything on the off equilibrium path by committing to give a large reward to the desired action. Thus, any threat of peer punishment cannot be credible, and collusion constantly forms. This assumption is true when the initiator does not have enough endowment to carry out the reward as promised.

Commitment gives the agents a much stronger ability to coordinate into corruption, so we have the following proposition:

Proposition 6. *Under assumption 3, in a fully connected communication network, for any number of agents $n \geq 2$, for all discount factor $\delta < 1$, there is no original equilibrium strategy that can make the agents exert any positive level of effort after the commitment stage.*

The proof is in the AppendixD2]AppendixD2 The intuition of the proof is that: If the principal can choose an equilibrium such that some players reject the commitment, then these players must be expecting strictly positive rewards for rejecting the commitment offer. However, the rewards can only come from someone who also rejects the commitment because once an agent commits to be corrupt, he will not give any transfer to anyone in the non-commitment group. However, suppose one player needs to give transfers to others while rejecting the commitment. In that case, such rejection cannot be incentive compatible because that player can always be better off accepting the commitment. In other work, the initiator commits to always bail out the reward-giving player if she is ever to be punished by the others. Thus, the previous rejectors cannot get the reward from the last player. By backward induction, everyone joins the commitment.

The commitment device gives players a solid ability to coordinate corruption. Similar to the voting for equilibrium selection case, the vulnerability of the effort provision equilibrium comes from the high connectivity of the department. If a receiver rejects the commitment, the initiator can circumvent this receiver and contact the rest of the department. So the principal can collapse any peer rewarding strategy designed by the principal. In such a case, the principal has no credible way to reward the rejecter so that everyone will join the commitment. [23]

Next, I show that it is still possible to deter corruption by commitment when the principal can limit the initial communication and punishment network.

Limited Communication and Deviation by Commitment The single ring supervision network can deter joint deviation by commitment. Like the previous section, we assume that a

[23] One thing to point out from the above proof is that: a joint commitment to described strategy is not the optimal strategy for the initiator. The initiator can get some positive transfer from the others, yet, everyone still agrees to join the commitment. However, since the principal cannot possibly deter a deviation by commitment, it is unclear what original strategy he would choose. Because whatever strategy he chooses, his payoff is the same. However, different original strategy matters for how much benefit that the initiator can get. So I will not discuss it in detail in that case.

receiver will share the contact information with the initiator if she joins the commitment plan. If she refuses to join the commitment, she will not share the contact information. In this section, I prove the following proposition:

Proposition 7. *Under the assumption 3 and the single ring supervision network, there exists an equilibrium selection function of the principal eq_p which can deter any deviation by commitment if there are at least six agents in the department.*

The proof is in the AppendixD3]AppendixD3 The intuition is similar to the proof of the proposition 3. When the initiator can only contact two other players, they serve as the "gatekeepers" and can shield the initiator from reaching the rest of the department. The "gatekeepers" could ask for massive bribery to join the commitment plan. The bribery is so large that the initiator is unwilling to give it. Threatening no-voters with punishment does not help either. In the single ring supervision network, no matter how many people join the commitment, they can only threaten two rejecters. So, in a large department, the principal can stop the deviation.

Also, it is essential to assume that the commitment group cannot transfer utility to the non-commitment group, as we can see in the following corollary.

Corollary 4. *The initiator can always form a joint commitment to no-effort if assumption 3 is removed so the commitment group can give transfer to any other player.*

Consider the following commitment of the initiator: any player who joins the commitment commits to no effort, no transfer, and no punishment. The initiator commits to fully compensate agents who are punished for not giving rewards to other rejecters. Thus all the receivers strictly prefer not to provide transfers.

In general, limiting the communication among players can significantly improve the robustness of equilibrium and deter the players from joint collusion. However, there are still limitations on how much it can achieve.

D2: Prood of Proposition 6

The proof is still under revision now. The intuition is the follow: For anyone to reject the

D3: Proof of Proposition 7

The assumption that the commitment group cannot give transfer to the non-commitment players significantly reduces the dimension of the analysis.

We can solve the bargaining stage of the game by backward induction. First, the commitment group can always get transfer from the non-commitment group by threat of punishment. The largest threat of punishment is 2.

1. When there is only one player rejects the commitment, its daily payoff is -2, which means, she gives 2 unit of transfer to the commitment group to avoid being punished.

2. When there are two players rejecting the commitment, each of them have a daily payoff of -1. Each of them gives the commitment group 1 unit of transfer to avoid being punished.

3. When there are two players rejecting the commitment, and one player in between, then each rejecter gets a daily payoff of 0 (punish the commitment group and get the transfer from the middle player), while the player in between gets -2. This is achieved by both rejecters punish the commitment group contact and minimax threat the player in between. If a rejecter deviates from punishing the commitment group, then the middle player would refuse to give her a transfer the next morning and punishing him instead in the afternoon. This is going to cost the deviated rejecter $-1\delta < 0$ So the deviation is not incentive compatible. Lastly, we only need to check that The middle player is indifferent between punishing the deviate and giving a transfer. So the future punishment is incentive compatible. When there are more player between the rejecters, the rejecter always pick a fight with the contingent commitment group members.

4. When there are two rejecters and two players in between, then each rejecter gets a payoff of 1, while both players in between gets payoff -2. Strategy is the same. This arguments goes through for more and more players between two rejecters. Each of the players in between gives the two rejecters 2 units of daily transfer, and this is maintained by the minimax payoff. The two rejecters only needs to pay the commitment group a transfer in total 2 units of payoff. Thus, each rejecter should get $\frac{2r-2}{2} = r - 1$ daily payoff, where r is the number of players between the two rejecters.

Let there be n players in the department, if the first receiver refuses to join the coalition, then if the second receiver also refuses, then each of them can get a payoff of $n - 3$. When $n \geq 6$, if the initiator wants to make the second receiver accept the commitment, the initiator needs to promise a daily reward of at least $n - 3 \geq 3$. This comes from the transfer from the non-commitment group (when there are only two rejecters out of the commitment), and the transfer that the initiator is willing to give (which is at most e_0, since this is on the equilibrium path). By making e_0 close to zero, the second receiver would never get enough rewards for rejection, so the second receiver also refuse to join the coalition.

Then, we consider the decision of the first receiver. If the first receiver reject the commitment, she gets $n - 3 = 3$ unit of payoff as we specified above. If the first receiver accept the offer, the initiator needs to promise the receiver a equilibrium result of at least 3 unit of payoff. However, this cannot be possible when e_0 is close to zero.

When $e_0 < 2$ is close to 2, the initiator can pay for that. But there would only be less than 1 unit of payoff left. If the second receiver rejects the offer, the third receiver would also do that, which would give both players a reward of 1. So, both the second and third receivers would reject the offer. If the second player cannot possible get 1 unit of payoff by accepting the offer. So, still rejection is inevitable.

When there are more players in the department, the second and the third receivers would get more reward by rejecting the offer. So, they will not join the commitment. This completes the proof. □

6.1 E1: Extension

To simulate the real-world, there might be several model extensions that are needed. 1. There is a cost of communication for each phone call. So there is a difference between the initiator calling each person himself and asking the receiver to do the thing replacing him. 2. Multiple initiators and the coordination of clans. 3. There will be repeated re-negotiation of the power.

There is a more general model about the cycle of power and regime change. Here is a verbal version of it. If we think about coordinating a upheaval in a country of billions of people, then the above extensions needs to be considered seriously. The people first is unsatisfied with the status quo and ask for a total reform. They have to get enough support so that their revolution can success. So, they go through the renegotiation process to claim the power by proposing an equilibrium that will benefits enough people so the revolution is going to be a success. The state, on the other hand, tries the best to block the communication.

As they obtain the power, the successful revolutionist group is exempt from any punishment and on the other hand, they punish those who previously objects to the revolution. However, as the new nation start to grow, there will be a serious of power rearrangement. A new class of people who has power will occur. This is done by a serious of renegotiation within departments of the state. The newly established regime would try its best to stop such deviation from happening by established another peer supervision structure. However, the communication network among those who have power has to grow overtime. There will be some point that the communication network is so complete, that the joint deviation to corruption also occurs.

Those who holds power gradually gets more power, which means they can initiate a joint deviation with fewer and fewer support. Naturally if there is ever a leader that does not value of being moral so much that it overweight the benefit of renegotiate to a more economically beneficial improving state for himself, then the new cycle begins. The new leader would choose a series of policies to acquire more power and suppress the power of his opponents. That is the birth of dictator and suppressive policies. Such policies triggers a new cycle of political regime change.

There are several ways to stop the cycle: the biggest initiator of a revolution is when the people in power take such a large share of the cake that those who are not in power fall short of subsistence, and they also know enough people fall short of the subsistence level that a rebel could be successful. So it is critical that the regime make sure not to overly claim the cake and leave something for the lower tire to survive.

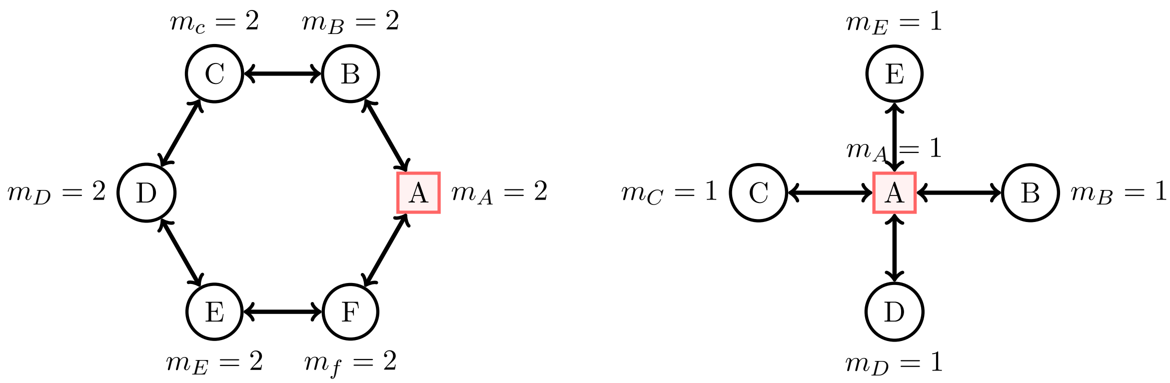

Figure 5: The number next to each node represents the player's bargaining power $m_i(S)$