A BEHAVIORAL APPROACH TO DURABILITY CHOICE, NEW-PRODUCT INTRODUCTIONS, AND PLANNED OBSOLESCENCE *

CHENYANG LI† MICHAEL WALDMAN‡ HAIMENG HESTER ZHANG§

November 3, 2022

Abstract

Observation of real-world markets suggests that many products are produced at below efficient built-in durability levels, and/or new products are introduced quickly which inefficiently reduces the useful life of durable products. Most of the prior literature on this subject explains these observations employing monopoly/market power models, but a number of the markets that exhibit these behaviors are competitive. We consider models in which consumers have time-inconsistent/present-biased preferences, as first put forth in the seminal analysis of Strotz (1955), and show that present-biased consumer preferences can lead to equilibrium durability below efficient levels and inefficiently quick new-product introductions, even in competitive markets. We also investigate circumstances in which market power aggravates these distortions. In addition to deriving these theoretical results, we relate our theory to recent regulatory changes in the light bulb industry, as well as to the behavior of the well-known Phoebus light bulb cartel of the 1920s and 1930s.

Keywords: durability, new-product introductions, present bias

JEL codes: D4, L4

*We thank Si Zuo for helpful discussions and Lakshita Sharma for research assistance.

†Department of Economics, Cornell University Uris Hall 451, Ithaca, NY 14853. Email cl2453@cornell.edu

‡Johnson Graduate School of Management, Cornell University, 323 Sage Hall, Ithaca, NY 14853, (607) 255-8631, mw46@cornell.edu

§IESE Business School, U-305, North Campus, C/Arnús i Garí, 3-7, 08034 Barcelona, Spain hzhang@iese.edu

1 INTRODUCTION

There are numerous examples of markets in which product durability seems below what is easily achievable. This could be because of new-product introductions that make used units obsolete as in the video game or fashion industries, or because the product has a low level of built-in durability as seems to have been the case for light bulbs prior to the recent regulatory changes. Previous theoretical literature on the subject has mostly focused on monopoly/market power models to explain the behavior. But many of the real-world examples where the behavior is observed seem closer to competition than to monopoly. In this paper, we explore the extent to which time-inconsistent/present-biased preferences, as first put forth by Strotz (1955) and extended, for example, by Lowenstein and Prelec (1992), Laibson (1997), and O'Donoghue and Rabin (1999), can explain inefficiently low levels of built-in durability and frequent new-product introductions in both competitive and monopoly settings.

Why would present-biased consumer preferences be important for durability choice and the frequency of new-product introductions. We know that, in the absence of the ability to commit, present-biased consumer preferences can lead to savings levels below efficient levels, as discussed initially by Strotz (1955). The main point of our paper is that purchasing a good with a longer useful lifetime can be similar to increasing savings in that it increases future consumption at the expense of current consumption. So, similar to present-biased preferences resulting in below efficient savings levels, such preferences can also result in inefficiency concerning built-in durability choice and the frequency of new-product introductions.

In the first part of our analysis, we consider an infinite-period model in which new and used units are perfect substitutes in consumption, where durability choice concerns the probability that a unit lasts an extra period. Consumers have present-biased preferences of the type modeled by Laibson (1997) and O'Donoghue and Rabin (1999), where we initially assume consumers are naïve meaning that a consumer in one period does not anticipate that she will exhibit present bias when making consumption decisions in future periods. Also, consumers in each period derive utility from the consumption of the durable good and from the consumption of "other goods" which we denote as the numeraire good.

In this model, given perfect competition, firms produce units characterized by the first-best durability level in the absence of present-biased preferences, but produce units with less durability when consumers exhibit present bias and are naïve. The logic is that more durable units cost more to produce which under perfect competition means they have a higher price. So by purchasing less durable units that cost less, a consumer can shift consumption from future periods to the current period. With time-consistent preferences, the incentive to shift consumption from the future to the present is determined by the discount factor common to all players in the game, with the result that firms produce and sell units with the efficient durability level. But when consumers have present-biased preferences and are naïve, then the incentive for consumers to move consumption from the future to the present exceeds that suggested by the common discount factor, and the result is a durability level below the first best.

We also consider equilibrium behavior given monopoly rather than perfect competition. Here we find results similar to what we found in the competitive case. That is, when consumers are characterized by time-consistent preferences, the monopolist chooses the first-best durability level. But when consumers exhibit present bias and are naïve, then the monopolist produces and sells units that are characterized by durability below the first best. The logic here is that, given present-biased consumers and after correcting for standard discounting, consumer willingness to pay in the current period for current-period services is higher than willingness to pay for these services in any previous period. As a result, a durable-goods monopolist can increase profits by producing less durable units since that increases the frequency with which current-period services are purchased in the current period.

We also compare equilibrium durability, given present-biased and naïve consumers, under perfect competition and monopoly. Here we find that, if the value consumers place on consuming the durable product in a given period is sufficiently high and/or the cost of producing the durable good is sufficiently low, then equilibrium durability is lower given monopoly than under perfect competition. That monopoly durability is lower when consumer value for consuming the durable good is high follows because the incentive for firms to reduce durability below the efficient level in the competitive case is independent of this value, while the incentive for a monopolist to reduce durability below the efficient level increases with this value. And the related result concerning the cost of producing the durable good follows from a related logic.

In our last analysis of this first model, we consider how results change when consumers are sophisticated rather than naïve, where a sophisticated consumer anticipates in each period that in making purchase decisions in later periods, she will exhibit present bias. Here we find that most of the qualitative results for the naïve consumer case continue to hold for sophisticated consumers. For example, it is still the case that, under both perfect competition and monopoly, the equilibrium durability level is below the efficient level. We do find, however, that the distortion in the case of perfect competition is smaller when consumers are sophisticated. In contrast, in the case of monopoly, behavior is the same in the sophisticated and naïve consumer cases.

In the second part of our analysis, we focus on how present-biased consumer preferences affect new-product introductions that make used units obsolete. In particular, we consider a three-period model in which there is exogenous technological progress over time, so a new product introduced in period two is of higher quality than period-one production, and a new product introduced in period three is of higher quality than a new product introduced in period two. Our main focus is whether present bias can result in a new product being introduced in period two, where this would not be the case in the absence of present bias. Note that in this model, given the three-period structure and the parameterizations we focus on, there is no difference between the naïve and sophisticated consumer cases.

Here we show that, given both perfect competition and monopoly, when consumers have time-consistent preferences, a new product is introduced in period two rather than period three when it is efficient to do so, i.e., when the introduction is such that the discounted change in gross consumer utility over periods two and three exceeds the discounted change in costs. In contrast, when consumers have present-biased preferences, under perfect competition there are parameterizations for which a new-product introduction occurs in period two rather than period three even when this condition is not satisfied.

We show that this result can occur given parameterizations in which a new-product introduction in period two stops a new product from being introduced in period three, due to the reduced incremental quality associated with a period-three introduction. The logic is that in period two present-biased consumers place a low value on consuming a higher quality product in period three, so a present-biased consumer cares little that a new-product introduction in the second period stops the introduction of an even higher quality new product in period three. The result is that in perfect competition with present-biased consumers, a new product may be introduced in period 2 when it is efficient to delay the new-unit purchase to period 3.

We also find that for the same set of parameterizations, a monopolist which faces present-biased consumers will sometimes choose to delay the new-product introduction to period 3 when it is efficient to introduce the new product in period 2. The logic here is that, after correcting for standard discounting, present-biased consumers have a higher willingness to pay for period-3 durable good services in period 3 than in period 2. This gives the monopolist an extra incentive to delay the new-product introduction until period 3, with the result that the monopolist sometimes introduces the new product in period 3 when it is efficient to introduce the new product in the second period.

We also consider parameterizations for which there is no period-three introduction independent of whether or not there is a period-two introduction. Here we find that, given present bias and perfect competition or monopoly, there is sometimes no new-product introduction after the first period when introducing a new product in period 2 would be efficient. The logic for this result follows from the basic feature of what it means to be present biased. That is, for these parameterizations the return to a new-product introduction in period two is higher gross utility in periods two and three from the consumption of a higher quality unit of the durable good. Present-biased consumers in period two, however, place little value on any extra utility associated with consuming a higher quality unit in period three. As a result, for these parameterizations time-consistent consumers can find it optimal to purchase a new higher quality product in period two, while present-biased consumers do not.

At the end of the paper, we discuss a potential application of our analysis – the light bulb industry. Our discussion of the light bulb industry includes a discussion of the well-known Phoebus light bulb cartel established in 1925 which lasted more than a decade. The cartel raised prices and reduced the life expectancy of the light bulbs produced from approximately 1800 hours to approximately 1200 hours. Although the evidence is not completely clear, some who have studied this historical episode argue that the cartel intentionally reduced the durability of the bulbs below the efficient level (see, for example, Krajewsi (2014)). As we discuss in more detail later, prior theories of reduced durability and planned obsolescence are not good explanations for the cartel's behavior. However, the analysis in Section III provides a solid theoretical foundation for why the cartel may have found it profitable to practice a type of planned obsolescence.

Overall, this paper contributes to the literature concerning real-world applications of the idea that in many settings consumers exhibit present bias. This includes various behaviors related to savings (see, for example, Strotz (1955)), procrastination (see, for example, O'Donoghue and Rabin (1999)), and borrowing at high interest rates on credit cards (see, for example, Meier and Sprenger (2010)). Given the importance of durability choice and new-product introductions in many markets, we feel that our results concerning how present bias affects behavior in durable-goods markets may be one of the more important applications of the present-bias idea.

The outline for the paper is as follows. Section II discusses the related literature. Section III presents and analyzes our model of built-in durability. Section IV presents and analyzes our model of new-product introductions. Section V discusses a potential application of our theoretical findings. Section VI provides concluding remarks.

2 LITERATURE REVIEW

This paper mostly contributes to two literatures: i) papers on durability choice, new-product introductions, and planned obsolescence; and ii) papers concerning real-world applications of the idea that individuals have present-biased preferences, especially the small set of papers related to durable-goods markets. We start by discussing the literature on durability choice, new-product introductions, and planned obsolescence, and then discuss the literature concerning present bias. See Waldman (2003) for a survey that discusses the former literature, and O'Donoghue and Rabin (2015) for a survey that discusses the latter.

In a series of influential papers in the early 1970s, Peter Swan showed that under a wide range of settings both perfect competition and monopoly result in the efficient choice of durability (see, for example, Swan (1970,1971) and Sieper and Swan (1973)) [1]. The basic idea is that, if consumers care only about the services provided by the durable product and not durability directly, then under both market structures firms choose the cost minimizing or socially optimal level of durability. He showed this result both when new and used units are perfect substitutes, and when units depreciate or deteriorate with age, but some number of used units is a perfect substitute for a new unit.

A number of papers investigate settings in which Swan's argument does not apply. Probably the two most important are monopoly/market power arguments due to Bulow (1986), Waldman

[1] Swan's papers improve on earlier analyses of the durability issue such as Kleiman and Ophir (1966), Levhari and Srinivasan (1969), and Schmalensee (1970).

(1996a), and Hendel and Lizzeri (1999a)[2]. In his 1986 paper, Bulow builds on Coase (1972) and Bulow (1982), and shows that a monopolist may reduce durability to partially avoid the costs of time inconsistency which arise when a durable-goods monopolist cannot commit to future prices or production levels. Waldman (1996a) and Hendel and Lizzeri (1999a) show that, if used units are lower quality than new units and durability choice affects the speed of quality deterioration, a durable-goods monopolist may choose durability below the efficient level in order to more effectively price discriminate across consumers with different valuations on product quality [3]. In contrast to these market power arguments, we focus on a behavioral approach concerning consumer preferences which can apply in both market power and competitive settings.

A related literature focuses on reasons why durable-goods producers may reduce the effective durability of a product by frequent introduction of new units that make used units obsolete. One argument in this literature, due initially to Waldman (1993,1996b) and Choi (1994), is that time inconsistency can cause a monopoly producer to practice this type of planned obsolescence, i.e., a durable-goods monopolist that sells its output will not internalize how new-product introductions affect the value of used units with the result being inefficiently frequent new- product introductions. Another argument concerns the role that fashion plays in signaling status, income or related attributes in games of social interaction (see, for example, Pesendorfer (1995)). Our approach, in contrast, depends neither on market power, nor the product being easily observable by other market participants, as is required in signaling games of social interaction [4].

As indicated, the paper also contributes to the literature on time-inconsistent/present-biased preferences, as originally put forth in the seminal analyses of Strotz (1955), Lowenstein and Prelec (1992), and Laibson (1997). We contribute to the part of the literature that employs the present-bias assumption to explain various real-world behaviors. In particular, we contribute to this part of the literature by focusing on the implications of present-biased preferences for behavior in durable-goods markets – in particular, the endogenous choice of durability either through built-in durability or new-product introductions that make used units obsolete. Since purchasing a durable good is similar to savings in that the behavior translates into purchasing a good used for consumption in a future period, it seems intuitive that present bias will be important for equilibrium durability choice given that it is important for understanding various aspects of savings behavior. We systematically investigate equilibrium implications.

Note that a few previous papers have considered the implications of present-biased preferences

[2]See Schmalensee (1979) for a survey of earlier literature investigating settings in which Swan's argument does not apply.

[3]See Anderson and Ginsburgh (1994) for an earlier related analysis. Also, note that this argument builds on the seminal analysis of Mussa and Rosen (1978) concerning how a monopolist chooses prices and qualities when selling a product line.

[4]A third argument, due to Grout and Park (2005), is that new-product introductions that make used units obsolete can arise in competitive settings to reduce problems due to adverse selection concerning secondhand-market trade. Our analysis abstracts away from any problems concerning adverse selection and secondhand-market trade since in our model consumers are all identical so secondhand-market trade is not a factor (also, in our model used-unit quality is not stochastic which further stops adverse selection from being an issue). The application we discuss in Section V, i.e., the light bulb industry, is characterized by little or no secondhand-market trade.

for behavior in durable-goods markets. For example, Nocke and Peitz (2003) focus on how present bias affects secondary markets for durable goods, Gill and Hayashi (2021) consider debt financing used to purchase durable goods when consumers are present biased, while Kang and Kang (2022) show how purchasing durable goods can be used as a commitment device by sophisticated consumers characterized by present bias. All of these papers take the degree of product durability as given rather than endogenously determined. We instead focus on the implications of present bias for equilibrium durability – whether built-in or determined by the speed of new-product introductions – which is a classic issue in the industrial organization literature.

3 PRESENT BIAS AND BUILT-IN DURABILITY CHOICE

In this section we consider how present bias affects equilibrium built-in durability choice given both perfect competition and monopoly. In the first subsection we present the model, while in the second we analyze the model under the two market structures given naïve consumers. In the third subsection we consider the sophisticated consumer case. Note that our analysis includes a comparison of durability choice across perfect competition and monopoly given present-biased consumers preferences.

A) The Model

We consider an infinite-period discrete-time model. Consumers are infinitely lived, which is also the case for the single firm in the monopoly case and multiple firms in the case of perfect competition. Within a period new and used units that are in good working order are perfect substitutes in consumption, where units can be different levels of durability. To be precise, θ denotes the durability level, where a unit produced in any period t has a probability $(1-\theta)$ of lasting a single period, a probability $\theta(1-\theta)$ of lasting two periods, a probability $\theta^2(1-\theta)$ of lasting three periods, etc. We also assume that there are no fixed costs of production, while the marginal cost of production increases with the durability of the unit. In particular, producing x units of durability θ, the cost equals $xc(\theta)$, where $c(0) = C > 0$, $c(1) = \infty$, $c'(0) = 0$, $c'(\theta) > 0$ for all $0 < \theta \leq 1$, and $c''(\theta) > 0$ for all $0 < \theta \leq 1$ [5].

There are N identical consumers, where consumer income each period is given by Y. Consumers spend their income each period purchasing either zero or one unit of the durable good, while remaining income is spent on purchasing the numeraire or other goods – we do not allow savings or borrowing (see the Conclusion for a discussion). We also assume that Y is sufficiently large that a consumer purchases a positive amount of the numeraire good in every period.

Utility for representative consumer i in period t, $\mu_{i,t}$, is given by equation (1)

[5] Throughout the analysis we also assume that the cost function is sufficiently convex that there is a unique equilibrium, given both perfect competition and monopoly, when consumers are time consistent. See the Appendix for details.

$$
\begin{aligned}
\mu_{i,t} &= b_{i,t}V + f(x_{i,t}) \\
s.t.\ \ w_{i,t} + x_{i,t} &= Y
\end{aligned}
\tag{1}
$$

In equation 1, $b_{i,t} = 1(0)$ when the consumer owns (does not own) a working unit of the durable good in period t, where the consumer can own a working unit by either purchasing a new unit in period t or by owning a working unit purchased in a previous period.[6] V is the consumer's gross utility from owning a working unit of the durable good in any period, $V > C$. xi,t is the number of units of the numeraire good consumed in period t, where $f(x_{i,t})$ captures the consumer's gross utility in period t from consumption of the numeraire good. We assume $f(x_{i,t}) = x_{i,t}$ in the main body of the paper. The utility function is thus quasi-linear. It gives us the clearest solutions. In the appendix, we also extend our model and allow more general utility function: $f(0) = 0$, $f(0) = 1$, $f'(x) > 0$ for all $x \geq 0$, and $f''(x) \leq 0$ for all $x > 0$. Most of the results still hold. In the budget-constraint equation, $w_{i,t}$ denotes the consumer's expenditure in period t purchasing the durable good.

In each period t, consumer i chooses behavior consistent with maximizing her perception of expected utility over the remainder of her lifetime. To be precise, in each period t consumer i maximizes $U_{i,t}$ which is given in equation (2).

$$
U_{i,t} = \mu_{i,t} + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-1}\mu_{i,t} \tag{2}
$$

δ is the discount factor, $0 < \delta < 1$, and including β means we allow for the possibility that consumers are present biased, where we incorporate present bias by employing the now standard assumption of hyperbolic discounting. In particular, we consider both the case in which consumers are time consistent, i.e., $\beta = 1$, and the case of present bias, i.e., $0 < \beta < 1$. Note that in the next subsection we assume consumers are naïve in the sense defined by O'Donoghue and Rabin (1999), i.e., in making decisions in each period t, consumers ignore the idea that in future periods their behavior will be present biased. O'Donoghue and Rabin (2015) argue that this is the more realistic assumption. Also, firms are not present biased which means for firms discounting is determined solely by the discount factor δ.

The timing of the game is as follows. At the beginning of each period, every consumer who owned a working unit in the previous period observes whether or not the unit will work this period. Every durable-goods producer then chooses a durability level for its output and a price. Each consumer then makes period-t purchases, where any income not spent on purchasing a unit of the durable good is spent on the numeraire good. Our focus is Markov Perfect equilibria.

[6] We do not allow secondhand-market trade, but that has no significant effect on equilibrium behavior given consumers are identical. See footnote 4 for a related discussion.

B) Analysis with Naïve Consumers

In this subsection, we analyze the model presented in the previous subsection given both time-consistent consumers and present-biased consumers who are naïve. We start by characterizing the first best. The first best in this case is the behavior that would be chosen by a social planner who decides production levels in each period, how production is allocated across consumers, and who maximizes the expected discounted value starting in period 1 of realized consumer gross utilities minus the costs of production.[7]

It is easy to show that for our model, independent of whether consumers are time consistent or present biased, every consumer is allocated a new durable unit in any period in which the consumer does not own a working durable unit at the beginning of the period, and durable units are produced at durability level θ^* which satisfies equation (3).

$$c'(\theta^*) = \delta c(\theta^*)/(1-\delta\theta^*) \tag{3}$$

Equation (3) captures that the first-best durability level is the durability level that minimizes the expected cost of having all consumers consume a unit of the durable good in every period. That is, since in the first best all consumers consume a unit of the durable good in each period, maximizing social welfare reduces to finding the durability level that minimizes the cost of achieving this result.

We first consider how the model works when consumers are time consistent.

Proposition 1. *When consumers are time consistent, i.e.,* $\beta = 1$*, then under both perfect competition and monopoly, all consumers consume a durable unit in each period and firms produce units of durability* θ^**. Also, i) and ii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price for a durable good in each period is* $c(\theta^*)$ *and all surplus is received by consumers.*
- *ii) Given monopoly, the new-unit price for a durable good in each period equals* $V(1 + \delta\theta^* + \delta^2\theta^{*2} + \dots)$ *and all the surplus is received by the monopolist.*

Proposition 1 demonstrates that our model is consistent with Swan's conclusion concerning durability choice. That is, when new and used units are perfect substitutes in consumption, then under both perfect competition and monopoly the firms will choose the durability level that minimizes the cost of producing the equilibrium flow of services. In other words, under both perfect competition and monopoly, the firms choose the socially-optimal durability level.

The next part of the analysis shows that this result depends on the assumption that consumers have standard or time-consistent preferences. That is, if consumers are present biased rather than

[7] In our analysis of the first best, we aggregate consumer utilities across periods setting $\beta = 1$. This is standard for models of hyperbolic discounting. This is also how we calculate consumer surplus in our present-bias analysis. See O'Donoghue and Rabin (1999) for a discussion.

having time-consistent preferences, then firms choose durability levels below rather than equal to the socially-optimal level. [8]

Proposition 2. *When consumers are present biased, i.e.,* $\beta < 1$*, and naïve, then under both perfect competition and monopoly, all consumers consume a durable unit in each period and firms produce units of durability* $\theta_{P,N'}$ *,* $\theta_{P,N'} < \theta^*$*, in the case of perfect competition, and* $\theta_{M,N'}$*,* $\theta_{M,N'} < \theta^*$*, in the case of monopoly. Also, i) and ii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price in each period is* $c(\theta_{P,N'})$*. Also, both consumer welfare and social welfare are below the values when* $\beta = 1$ *because of the distortion in the durability level (while producer welfare is unchanged since it equals zero in each case).*
- *ii) Given monopoly, the new-unit price in each period equals* $V[1 + \beta(\delta\theta_{M'} + \delta^2\theta_{M'}^2 + \delta^3\theta_{M'}^3 + \ldots)]$*. In this case, monopoly profit is below the value when* $\beta = 1$*, while consumer surplus can be higher or lower than when* $\beta = 1$*.*

The main result in Proposition 2 is that when consumers are present biased and naïve, given both perfect competition and monopoly, firms produce units that are less durable than in the first best, or equivalently, less durable units than when consumers have time-consistent preferences. The logic for this result in the perfect competition case is as follows. More durable units cost more to produce, which in the case of perfect competition translates into a higher new-unit price. The return to having more durable units is a less frequent need to buy replacement units which, in turn increases future consumption of the numeraire good. But a present-biased consumer places less value on future consumption of the numeraire good, so has a smaller willingness to pay for incremental increases in durability which translates into a lower equilibrium durability level.

The logic for the monopoly result is related. The monopolist charges consumers their willingness to pay for a new unit given they do not currently own a working unit. Present bias means consumers value the future less and are thus less willing to pay for incremental increases in durability. The result is lower durability than the equilibrium durability given time-consistent consumers. Another way to look at this result is that, because of present bias and after correcting for standard discounting, consumer willingness to pay for current period services is higher in the current period than in any earlier period. Reducing durability increases the probability that current period durable-goods services are purchased in the current period, which takes advantage of the higher willingness to pay and thus increases monopoly profitability.

The proposition also contains a number of other interesting results. For example, present bias hurts consumers but not firms under perfect competition, while under monopoly present bias hurts the monopolist and has an ambiguous effect on consumer surplus. The result concerning perfect competition follows given that the distortion concerning the equilibrium durability choice hurts social welfare, and we know that under perfect competition all surplus goes to the consumers. That

[8] In our monopoly analysis of the naïve consumer case, we assume that in each period each consumer believes that in future periods every consumer will not exhibit present bias.

present bias hurts monopoly profit follows given that present bias reduces consumers' willingness to pay and thus monopoly profitability, holding durability fixed, while the reduction in durability due to present bias further reduces monopoly profitability.

Our next analysis concerns a comparison of the equilibrium durability choices under perfect competition and monopoly given present-biased consumers who are naïve. This comparison is important for our discussion of the Phoebus cartel that appears later.

Corollary 1 (to proposition 2)**.** *Suppose consumers are present biased, i.e., $\beta < 1$, and naïve. Then, holding everything fixed other than $V(C)$, there exists a value $V'(C')$ such that, if $V > V'$ ($C < C'$), the equilibrium durability level given monopoly is below the equilibrium durability level given perfect competition.*[9]

Suppose consumers are present biased and naïve. Under perfect competition, as discussed above, the incentive to reduce durability is related to the low cost in terms of current utility a consumer perceives associated with having to purchase a new durable unit in a future period. In contrast, under monopoly the incentive to reduce durability is the higher value a consumer places on consuming a working durable unit in a future period when that future period actually arrives. The former is independent of V, while the latter increases with V. The result is that, if consumers are present biased and V is sufficiently high, the equilibrium durability level is lower under monopoly than under perfect competition. Note that a related logic explains why when C is sufficiently low equilibrium durability is again lower under monopoly than under perfect competition.

C) Present Bias with Sophisticated Consumers

In this subsection we analyze the model given consumers are present biased as above, but now we assume the consumers are sophisticated rather than naïve. The definition of a sophisticated consumer is that in each period the consumer correctly anticipates that in future periods she will exhibit present bias. Also, the first point to note is that efficient durability is independent of whether consumers are naïve or sophisticated. So the efficient durability level is still given by θ^* and is still characterized by equation (3) above.

We now consider equilibrium durability given present bias and sophisticated consumers. Note that in the analysis that follows we focus on stationary Markov Perfect equilibria.[10]

Proposition 3. *When consumers are present biased, i.e., $\beta < 1$, and sophisticated, all consumers consume a durable unit in each period and firms produce units of durability $\theta'_{P,S}$, $\theta_{P,N'} < \theta_{P,S'} < \theta^*$, in the case of perfect competition, and $\theta_{M,S'}$, $\theta_{M,N'} = \theta_{M,S'} < \theta^*$, in the case of monopoly. Also, i) and ii) further describe equilibrium behavior.*

[9]For this analysis, we assume $c(\theta) = C + g(\theta)$, where $g(0) = 0$, $g'(0) = 0$, $g'(\theta) > 0$ for all $0 < \theta \leq 1$, and $g''(\theta) > 0$ for all $0 < \theta \leq 1$.

[10]There may be non-stationary Markov Perfect equilibria in the case of sophisticated consumers. We follow O'Donoghue and Rabin (2002) and Acharya et al. (2022) in focusing on stationary Markov Perfect equilibria.

- *i) Given perfect competition, the new-unit price in each period is* $c(\theta P, S')$. *Also, both consumer welfare and social welfare are below the value when* $\beta = 1$ *but above the value in the naïve consumer case, because there is a distortion but it is smaller than in the naïve consumer case (while producer welfare is unchanged since it equals zero in all cases).*

- *ii) Given monopoly, behavior is described by ii) of Proposition 2.*

Proposition 3 tells us that, in the case of perfect competition, the equilibrium durability level under present bias with sophisticated consumers is below the efficient level, but the distortion is smaller than when consumers are naïve. However, in the case of monopoly, the sophisticated consumer equilibrium is identical to the naïve consumer equilibrium. The logic for the result concerning perfect competition is that sophisticated consumers realize they will exhibit present bias in the future and thus that in the future they will purchase units with inefficiently low durability levels. Anticipating this, they purchase units with a higher durability level than naïve consumers who believe future choices will be efficient. The logic for the monopoly result is that both naïve and sophisticated consumers anticipate that the monopolist will extract all future surplus, so willingness to pay in the current period for units of a given durability level is independent of whether a consumer is sophisticated or naïve. And the result is the same durability level for both sophisticated and naïve consumers.

In summary, consumers with present-biased preferences purchase units with inefficient low built-in durability under both perfect competition and monopoly. In the monopoly case the degree of distortion is independent of whether consumers are naïve or sophisticated. However, in the case of perfect competition, the distortion is larger when consumers are naïve.

4 PRESENT BIAS AND NEW-PRODUCT INTRODUCTIONS

In this section we present our model of new-product introductions, where we investigate how present bias affects the speed at which new-product introductions take place. The first subsection presents the model, while in the second we analyze the model both when consumers have time-consistent preferences and when consumers have present-biased preferences. Our main finding is that, in the case of perfect competition, present bias can cause a new-product introduction to occur earlier than is efficient and similarly earlier than what happens when consumers have standard preferences.

A) The Model

We consider a three-period model in which there are N identical consumers who are alive all three periods. Consumers derive utility from the consumption of a durable good and the consumption of other goods which are referred to as the numeraire good. Utility for representative consumer i in period t, $\mu_{i,t}$, is given by equation (4).

(4)
$$\mu_{i,t} = vb_{i,t}Q_{i,t} + x_{i,t}$$
$$s.t. w_{i,t} + x_{i,t} = Y$$

In equation (4), $b_{i,t} = 1$ ($b_{i,t} = 0$) when the consumer owns (does not own) a unit of the durable good in period t, where the consumer can own a unit by either purchasing a new unit in period t or by owning a unit purchased in a previous period.[11] $Q_{i,t}$is the quality of the unit owned by the consumer in period t ($Q_{i,t} = 0$ if the consumer does not own a unit) and v is the value consumers place on a unit of quality of the durable good. Also, in this model we assume that the marginal utility from the consumption of other goods, i.e., the numeraire good, is constant and equal to one rather than assuming decreasing marginal utility for the consumption of the numeraire good within a period. This assumption is not essential, but serves to simplify the algebra and make the basic logic of our results easier to follow. In the budget-constraint equation, $w_{i,t}$ denotes the consumer's expenditure in period t purchasing the durable good and Y is the per period income.

In each period t, consumer i chooses behavior consistent with maximizing her perception of expected utility over the remainder of her lifetime. To be precise, in period 1 consumer i maximizes $U_{i,1}$ which is given in equation (5), while in period 2 the consumer maximizes $U_{i,2}$ which is given in equation (6) (in period 3 the consumer maximizes $\mu_{i,3}$ which is given in equation (4)).

(5)
$$U_{i,1} = \mu_{i,1} + \beta \sum_{\tau=3}^{3} \delta^{\tau-1} \mu_{i,t}$$

(6)
$$U_{i,2} = \mu_{i,2} + \beta\delta\mu_{i,3}$$

δ , $0 < \delta < 1$, is the discount factor and, as before, including β means we allow for the possibility that consumers are present biased and are characterized by hyperbolic discounting. Also, as before, $\beta = 1$ means the consumers have time-consistent or standard preferences, while $\beta < 1$ means that consumers have time-inconsistent or present-biased preferences.

We assume there is exogenous technological progress. To be precise, a product introduced and sold in period t is of quality Q_t, where our assumption of exogenous technological progress means $Q_3 > Q_2 > Q_1$. Since in any period a consumer only receives utility from consuming a single unit of the durable good, a consumer who purchases a new unit in any period t who already owns a used unit will sell the used unit for scrap. We assume the scrap value for a unit of the durable good is z, $z \geq 0$, i.e., the scrap value is independent of the quality of the used unit.[12]

[11] As in the previous model, we do not allow secondhand-market trade, but that has no significant effect on equilibrium behavior given consumers are identical.

[12] Introducing a scrap value in Section III's model would not change the qualitative results. We decided not to

Note that in our model units are perfectly durable. For example, a unit produced in period 1 is of quality Q_1 whether consumed in period 1, period 2, or period 3. We could instead assume that quality declines with the age of the unit, but the qualitative results would be unchanged, although the logic driving the results would be less transparent. Thus, in order to keep the intuition for the results easier to follow, we assume that units are perfectly durable. As in the earlier model, we also assume no borrowing or savings.

We consider both the case of monopoly and the case of perfect competition. In the case of perfect competition firms are identical, where a firm can produce durable units in each period at constant marginal cost c, $c > z$, and no fixed cost. The assumption $c > z$ tells us that it is not profitable to produce units of the durable good that are immediately scrapped. We also assume $vQ_1 > c$ which ensures that all consumers purchase a new unit in period 1. Firms are characterized by the same discount factor δ as are consumers. In the case of monopoly everything is the same except there is a single producer rather than multiple identical producers. Also, our focus in this section is Subgame Perfect Nash equilibrium.

As a final point concerning the setup of the model, because of the three-period nature of the model and the parameter restriction $vQ_1 > c$ which ensures that consumers purchase a durable unit in period 1, equilibrium behavior in the present-bias case is independent of whether consumers are naïve or sophisticated. So, in the analysis that follows, we do not specify whether consumers are naïve or sophisticated when we consider present bias.

4.1 B) Analysis

We begin our analysis by focusing on parameterizations for which it is efficient for a new product to be introduced in either period 2 or period 3, but not both periods. The parameter restriction that ensures this is $v(Q_3-Q_1) > c-z > v(Q_3-Q_2)$. The first part of this expression, $v(Q_3-Q_1) > c-z$, tells us that it is efficient to introduce a new product in period 3 if a new product was not introduced in period 2, while $c - z > v(Q_3 - Q_2)$ tells us that it is not efficient to introduce a new product in period 3 if a new product was introduced in period 2. The focus in investigating this part of the parameter space is whether the new-product introduction occurs in the efficient period, and also how present bias affects the date and the efficiency of the new-product introduction.

Let us start by considering first-best behavior for these parameterizations. If consumers purchase new units in period 2, then the increase in social welfare over the three periods relative to new units being purchased neither in period 2 or period 3 is denoted Δ_2 and is given in equation (7).

$$\Delta_2 = \delta[v(Q_2 - Q_1) + z - c] + \delta^2 v(Q_2 - Q_1) \tag{7}$$

introduce a scrap value for that model since the application we discuss in Section V is not characterized by a scrap value.

If instead the new product is introduced in period 3, then the increase in welfare, denoted $\Delta 3$, is given in equation (8).

$$\Delta_3 = \delta^2[v(Q_3 - Q_1) + z - c] \tag{8}$$

Equations (7) and (8) yield that, holding the other parameters fixed, there exists a value Q_2^*, $Q_2{*} = \delta(Q_3 - Q_1) + Q_1 + [(1-\delta)(c-z)/v]$, such that it is efficient to introduce the new product in period 2 when $Q_2 > Q_2^*$ and to introduce a new product in period 3 when $Q_2 < Q_2^*$. [13]

We next consider behavior for these parameterizations when consumers have standard or time-consistent preferences.

Proposition 4. *Suppose $v(Q_3 - Q_1) > c - z > v(Q_3 - Q_2)$. If consumers are time consistent, i.e., $\beta = 1$, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, all consumers then purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_2^*$ ($Q_2 < Q_2^*$). Also, i), ii), and iii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price for a durable good in each period is c and all surplus is received by consumers.*
- *ii) Given monopoly and $Q_2 > Q_2^*$, the price for a new unit in period 1 equals $(1+\delta+\delta^2)vQ_1$, while the price for a new unit in period 2 equals $(1+\delta)v(Q_2 - Q_1) + z$. Also, all surplus is received by the monopolist.*
- *iii) Given monopoly and $Q_2 < Q_2^*$, the price for a new unit in period 1 equals $(1+\delta+\delta^2)vQ_1$, while the price for a new unit in period 3 equals $v(Q_3 - Q_1) + z$. Also, all surplus is received by the monopolist.*

Proposition 4 tells us that when consumers have time-consistent preferences, then the new-product introduction occurs in the efficient period. The logic is as follows. In this model, given time-consistent preferences, consumers receive all the surplus under perfect competition, while the single firm receives all the surplus in the monopoly case. Given time-consistent preferences and perfect competition, since consumers receive all the surplus and current preferences match long-run utility which is used to define efficient behavior, consumers have an incentive to purchase new units at the efficient date. Also, a similar logic explains why monopoly behavior is efficient with the difference being in that case it is the monopolist that captures all of the surplus.

Why would present bias make a difference in this case? For the parameterizations we are currently focused on, if new units are purchased in period 2, then new units are not purchased in

[13] It is possible that the other parameters are such that Q_2^* violates the parameter restriction $c > v(Q_3 - Q_2)$. For these parameterizations, it is efficient to introduce a new product in period 3 rather than period 2 for all allowable values for Q_2.

period 3. We start with the case of perfect competition. By purchasing new durable units in period 2, consumers give up the period-3 utility due to the incremental quality associated with consuming a period-3 new unit rather than a period-2 new unit, i.e., $v(Q_3 - Q_2)$. But with present bias, in period 2 consumers undervalue this potential extra period-3 utility. The result is that, in the case of perfect competition, a new product is produced and sold in period 2 more often than is efficient.

Now consider the same set of parameterizations in the case of monopoly and present bias. Due to present bias, consumer willingness to pay for period-3 utility associated with consuming the durable good in that period is higher in period 3 than in period 2. This gives the monopolist an extra incentive to not sell new units in period 2 and instead delay selling new units until period 3. Thus, in contrast to the case of perfect competition, in the monopoly case a new product is produced and sold in period 2 less often than is efficient.

In Proposition 5 we formalize these results.

Proposition 5. *Suppose $v(Q_3 - Q_1) > c - z > v(Q_3 - Q_2)$. If consumers are present biased, i.e., $\beta < 1$, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, in the case of perfect competition (monopoly) there exists a value $Q_{2,P'}$ ($Q_{2,M'}$), $Q_{2,P'} < Q_2^*$ ($Q_{2,M'} > Q_2^*$), such that all consumers then purchase a new durable unit in period 2 and scrap the used unit if $Q_2 > Q_{2,P'}$ ($Q_2 > Q_{2,M'}$), while consumers purchase a new durable unit in period 3 and scrap the used unit if $Q_2 < Q_{2,P'}$ ($Q_2 < Q_{2,M'}$). Also, i), ii), and iii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price for a durable good in each period is c and all the surplus is received by consumers, where social welfare and consumer welfare are both below (equal to) the values when $\beta = 1$ if $Q_{2,P'} < Q_2 < Q_2^*$ ($Q_2 < Q_{2,P'}$ or $Q_2 > Q_2^*$).*

- *ii) Given monopoly and $Q_2 > Q_{2,M'}$, the price for a new unit in period 1 equals $(1 + \delta\beta + \delta^2\beta)vQ_1$, while the price for a new unit in period 2 equals $(1 + \beta\delta)v(Q_2 - Q_1) + z$. Also, the surplus is shared between the consumers and the monopolist, where the consumer surplus is higher than when $\beta = 1$and monopoly profit is lower than when $\beta = 1$.*

- *iii) Given monopoly and $Q_2 < Q_{2,M'}$, the price for a new unit in period 1 equals $(1 + \delta\beta + \delta^2\beta)vQ_1$, while the price for a new unit in period 3 equals $v(Q_3 - Q_1) + z$. Also, the surplus is shared between the consumers and the monopolist, where the consumer surplus is higher than when $\beta = 1$ and monopoly profit is lower than when $\beta = 1$.*

In addition to showing that present bias has a different effect on the incentive to distort the date of the new-product introduction depending on whether the market structure is perfect competition or monopoly, it is interesting to note that monopoly does not result in all surplus going to the monopolist (this was also true in Proposition 2). That is, even though the monopolist charges consumers their willingness to pay each period, given present bias, consumers receive positive surplus in equilibrium. This is because, even after correcting for discounting, consumer willingness to pay in any period t for services in a period t', t'>t, is in fact below the value the consumer actually derives from these services when t' actually arrives.

In the next part of the analysis, we focus on a different part of the parameter space. In particular, we now consider parameterizations for which it is never efficient to introduce a new product in period 3, but Q_3 is sufficiently high that it would be efficient to introduce a new product in period 2 if Q_2 is sufficiently close to Q_3, i.e., $(1+\delta)v(Q_3-Q_1) > c-z > v(Q_3-Q_1)$. Note that the closer δ is to one, i.e., there is little standard discounting, the larger the range of parameterizations that satisfy this condition.

We again start by considering first-best behavior. If consumers purchase new units in period 2, then the increase in social welfare over the three periods relative to new units being purchased neither in period 2 nor period 3 is again denoted δ^2 and is given in equation (7). Equation (7) yields that, holding the other parameters fixed, there exists a value $Q_2^{**} = Q_1 + [(c-z)/(1+\delta)v]]$, such that it is efficient to introduce the new product in period 2 when $Q_2 > Q_2^{**}$ and not to introduce a new product in period 2 when $Q_2 < Q_2^{**}$.

We now show that standard or time-consistent preferences yield efficient outcomes for these parameterizations.

Proposition 6. *Suppose $(1+\delta)v(Q_3-Q_1) > c-z > v(Q_3-Q_1)$. If consumers are time consistent, i.e., $\beta = 1$, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, all consumers then purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q_2^{**}(Q_2 < Q_2^{**})$. Also, i), ii), and iii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price in each period is c and all surplus is received by consumers.*
- *ii) Given monopoly and $Q_2 > Q_2^{**}$, the price for a new unit in period 1 equals $(1+\delta+\delta^2)vQ_1$, while the price for a new unit in period 2 equals $(1+\delta)v(Q_2-Q_1)+z$. Also, all surplus is received by the monopolist.*
- *iii) Given monopoly and $Q_2 < Q_2^{**}$, the price for a new unit in period 1 equals $(1+\delta+\delta^2)vQ_1$. Also, all surplus is received by the monopolist.*

As before, when consumers have standard preferences, choices concerning when a new product is introduced are efficient. This is because consumers receive all the surplus in the case of perfect competition and thus have an incentive in that case to make efficient new-product purchases. On the other hand, with monopoly the firm receives all the surplus, and thus in that case the firm has an incentive to introduce new products in an efficient fashion.

We now consider this set of parameterizations when consumers are present biased.

Proposition 7. *Suppose $(1+\delta)v(Q_3-Q_1) > c-z > v(Q_3-Q_1)$. If consumers are present biased,, i.e., $\beta < 1$, then under both perfect competition and monopoly, all consumers purchase* *a durable unit in period 1. In addition, holding all other parameters fixed, in the case of perfect competition (monopoly) there exists a value* $Q_{2,P''}$ *(* $Q_{2,M''}$ *),* $Q_{2,P''} > Q_2^{**}$ *(* $Q_{2,M''} > Q_2^{**}$ *), such that all consumers purchase a new durable unit in period 2 and scrap the used unit if* $Q_2 > Q_{2,P''}$ *(* $Q_2 > Q_{2,M''}$ *), while consumers do not purchase a new durable unit in either period 2 or period 3 if* $Q_2 < Q_{2,P''}$ *(*$Q_2 < Q_{2,M''}$*). Also, i), ii), and iii) further describe equilibrium behavior.*

- *i) Given perfect competition, the new-unit price in each period is c and all surplus is received by consumers, where social welfare and consumer welfare are both below (equal to) the values when* $\beta = 1$ *if* $Q_2^{**} < Q_2 < Q_{2,P''}$ *(* $Q_2 < Q_2^{**}$ *or* $Q_2 > Q_{2,P''}$*).*

- *ii) Given monopoly and* $Q_2 > Q_{2,M''}$*, the price for a new unit in period 1 equals* $(1 + \delta\beta + \delta^2\beta)vQ_1$*, while the price for a new unit in period 2 equals* $(1 + \beta\delta)v(Q_2 - Q_1) + z$*. Also, the surplus is shared between the consumers and the monopolist, where the consumer surplus is higher than when* $\beta = 1$ *and monopoly profit is lower than when* $\beta = 1$*.*

- *iii) Given monopoly and* $Q_2 < Q_{2,M''}$*, the price for a new unit in period 1 equals* $(1 + \delta\beta + \delta^2\beta)vQ_1$*. Also, the surplus is shared between the consumers and the monopolist, where the consumer surplus is higher than when* $\beta = 1$ *and monopoly profit is lower than when* $\beta = 1$*.*

The proposition tells us that for these parameterizations, given both perfect competition and monopoly, present bias results in an incentive not to introduce a new product in period 2. The logic here stems from the basic characteristic of a present-biased consumer. That is, in each period a present-biased consumer places little weight on utility in future periods. Given that for these parameterizations there is no period-3 new product introduction independent of whether there is a new-product introduction in period 2, the effect of introducing present bias is to reduce the value consumers place in period 2 on owning a higher quality durable unit in period 3. This, in turn, reduces consumer willingness to pay for a new durable unit in period 2. The result is that, under both perfect competition and monopoly, there are parameterizations in which a new product is not sold in period 2 when such sales would be efficient.

In summary, in our three-period model, present bias sometimes results in a new product being introduced earlier than is efficient. In particular, this arises when there is perfect competition and the choice is whether to purchase a new unit in period 2 or a new unit in period 3. However, in other cases present bias results in later than efficient rather than earlier than efficient introduction of a new product. For example, for the parameterizations for which perfect competition results in an incentive for new product introduction earlier than the efficient date, monopoly results in an incentive for later rather than earlier new-product introductions.

5 AN APPLICATION: THE LIGHT BULB INDUSTRY

The light bulb industry is a possible application of our analysis – specifically, the analysis concerning built-in durability in Section III. In the first subsection we discuss recent light bulb regulation, while in the second we discuss the Phoebus cartel. Note that a discussion of the general history of light bulbs can be found in Brox (2010), a recent discussion of US light bulb regulation can be found in Tabuchi (2022), while the Phoebus cartel and its behavior concerning the durability of light bulbs produced can be found in Krajewski (2014).

A) Light Bulb Regulation

Until relatively recently, most light bulbs, especially those used in homes, were incandescent light bulbs. This technology is typically associated with Thomas Edison who began serious research on the topic in the 1870s, although the basic technology was discovered long before Edison's work. The basic technology of an incandescent light bulb is that a wire filament is heated until it glows. There are many advantages of incandescent light bulbs, but one disadvantage is that they are energy inefficient relative to a number of other light bulb technologies. Specifically, a typical incandescent bulb converts less than five present of the energy produced into visible light, while other technologies such as fluorescent bulbs and LED bulbs are much more energy efficient.

In the last few decades the light bulb market has changed in a dramatic fashion. Numerous countries have passed regulations that have disadvantaged the production and use of incandescent light bulbs. These regulations typically do not directly make it illegal to manufacture or purchase incandescent bulbs, but rather the regulations focus on energy efficiency. However, given that incandescent bulbs are not energy efficient, the result of the regulations is that incandescent bulbs have gradually exited the market. For example, in the US the Energy and Independent and Security Act of 2007 was enacted that imposed energy efficiency requirements for many types of bulbs, and more recently the Biden administration reversed Trump administration policies with the result that the sale of most incandescent bulbs will be prohibited by the middle of 2023. The important point here is that the time series of market shares makes it clear that incandescent bulbs, despite their energy inefficiency, were not fully exiting the market in the absence of regulation.

Light bulbs would seem to fit the type of product analyzed by Swan in his series of influential papers in the 1970s. That is, the service flow from new and used bulbs are similar if not identical, so according to Swan a competitive industry (as well as a monopoly) should produce bulbs with the efficient durability level. But this argument seems inconsistent with the evidence concerning how regulation in this industry has actually worked.

Consider the cost of using incandescent bulbs today versus the cost of using LED bulbs. The LED bulbs are clearly more expensive in terms of their initial purchase price, but LED bulbs last significantly longer and use less energy. For example, Hutton Power and Light (2019) reports estimates of the cost of using two bulbs of similar brightness – a 60 watt incandescent bulb and a 12 watt LED bulb – employing the average cost of electricity in Virginia in 2019. In the article the authors calculate that it would cost \$93 in total to use incandescent bulbs to produce 1,000 hours of light per year over a ten-year period, while using LED bulbs to produce the same amount of light would cost less than \$20. That is, despite the fact that the typical LED bulb is more expensive, that the bulb lasts longer and uses less energy makes LED lighting significantly cheaper.

In other words, despite the idea that standard theory due to Swan suggests that in this type of market regulation is not needed to achieve an efficient level of durability, regulation aimed at improving energy efficiency seems to have moved the market to a more efficient outcome in terms of product durability. As discussed earlier, the main prior arguments concerning limitations of Swan's argument such as those found in Bulow (1986), Waldman (1996a) and Hendel and Lizzeri (1999a) do not seem to explain this outcome. Alternatively, there were initially concerns that the light from LED bulbs and bulbs employing other alternative technologies were not as pleasing/high quality as the light emitted from incandescent bulbs. But those disadvantages seem to have been short lived and the quality of light produced by LED bulbs is now considered similar in quality to that of incandescent bulbs.

The fact, however, that regulation resulted in more efficient product durability is consistent with Section III's model. In that analysis, the presence of present-biased consumers causes durability to not be efficient even when the market is perfectly competitive. So regulation that forces firms to abandon an inefficient technology can both improve durability efficiency, as well as increase both consumer and social welfare. Note that we are not arguing that we have definitive proof that the correct explanation for why regulation in the light bulb industry improved durability efficiency is that consumers are present biased. Perhaps, for example, the fact that the industry was competitive slowed the innovation process, although that does not seem to be a general feature of competitive industries. We do, however, find it of interest that the effect of the regulation in this industry seems consistent with what our theoretical model predicts.

B) The Phoebus Cartel

In December 1924, the Phoebus cartel, or formally the Phoebus S.A. Compagnie Industrielle pur le Développement de l'Ēclairage was created in Geneva, Switzerland. All major light bulb manufacturers in the world, including Germany's Osram, the Netherlands' Philips, and France's Compagnie des Lampes, were its members. General Electric was represented by its British subsidiary, International General Electric, as well as the Overseas Group, which consisted of its subsidiaries in Brazil, China, and Mexico. Other members included Hungary's Tungsram, the United Kingdom's Associated Electrical Industries, and Japan's Tokyo Electric.

The Phoebus cartel exercised its market power through strict quantity controls. The cartel divided the world market into national and regional zones, and assigned a sales quota to each of its member companies. Companies that exceeded their quotas were fined. While the Phoebus cartel did not directly fix prices, the quantity controls allowed it to maintain stable prices over time despite falling manufacturing costs. The strictly enforced quota system ensured that the cartel was not subject to any commitment issue concerning durable goods production and falling prices over time. In other words, consumers would not have anticipated that the cartel would reduce prices over time since the quotas allowed the cartel to avoid such an outcome. Also, the light produced by an incandescent light bulb typically does not dim in a significant way during the lifetime of the bulb. As a result, the services received by a consumer in any period from a new bulb were arguably identical or close to identical to the services from a used bulb of a similar design.

As discussed earlier, the absence of the commitment issue and no reduction in the quality of the services provided as a bulb ages suggest that arguments concerning reduced built-in durability due to Bulow (1986), Waldman (1996a), and Hendel and Lizzeri (1999a) do not apply in this setting. Rather, existing theory due to Swan (1970,1971) suggests that the cartel should have produced bulbs of the efficient durability level, and also that the formation of the cartel should have had no impact on the durability of the bulbs produced. But, in fact, the cartel significantly reduced durability over time and the behavior is sometimes described as the first successful implementation of planned obsolescence in modern manufacturing.

Before the creation of the Phoebus cartel, the average lifetime of an incandescent bulb was approximately 1,800 hours, while from 1925 to 1934 after the creation of the cartel the average lifetime decreased steadily to approximately 1200 hours. This reduction in durability was achieved by the cartel through rigorous research, close monitoring, and strict enforcement.

Documents found in the corporate archives in Berlin of the cartel member Osram demonstrate that this reduction in durability was intentional on the part of the cartel, and strictly enforced by the cartel. Over time cartel members modified the filament and adjusted the current or voltage in order to decrease the average lifetime of the bulbs, where the cartel standard was adjusted over time to achieve a steadily decreasing durability. In order to enforce these changes, each factory bound by the cartel agreement was required to send samples of its bulbs to a central testing laboratory in Switzerland, where the bulbs were tested to see whether they met the cartel standards. If a factory's bulbs were found to last shorter or longer than the cartel standard, the factory was required to pay a substantial fine.

There are conflicting views concerning the motivation for the decreasing over time durability of the light bulbs enforced by the cartel. There is no evidence that the decreased durability lowered production costs. The cartel members, as well as a British government commissioned study, argued that the motivation for the reduction in durability was to produce a higher quality and brighter bulb. [14] But others who have studied the episode, partially basing their conclusions on quotes from top management of a cartel member, argue that independent of any change in quality, the reduction in durability was an important goal of the cartel because very long lifetimes served to reduce profitability.[15]

Suppose that reduced durability was indeed an important motivation for the changes in light bulb production induced by the cartel. Is there a plausible economic theory that would explain the

[14] See The Monopolies and Restrictive Practices Commission (1951).

[15] For example, Krajewski (2014) reports that after discovering an instance where some members attempted to secretly introduce a longer lasting bulb, Anton Philips who was head of Philips warned that "after the very strenuous efforts we made to emerge from a period of long lifetimes, it is of the greatest importance that we do not sink back into the same mire... supplying lamps that will have a very prolonged life."

behavior? As argued, lack of commitment was not an important factor in this market, so Bulow's 1986 argument concerning reduced durability employed to avoid time-inconsistency problems concerning production levels does not seem relevant. Similarly, because services from a bulb do not reduce substantially as the bulb ages, the argument found in Waldman (1996a) and Hendel and Lizzeri (1999) concerning reduced durability resulting in more effective price discrimination also does not seem to apply. In contrast, Section III's model provides a clear explanation for why the cartel might have had an incentive to reduce the expected lifetime of the bulbs. In that model, because consumers are identical, there is no Coase/Bulow time-inconsistency problem in which a monopolist would want to lower price over time and hurt its own profitability. Further, the utility derived in a period from a unit of the durable good does not depend on the good's age as long as the unit is in good working order which matches the cartel's situation. But the analysis shows that, if consumer preferences are present biased, then a monopolist would want to produce less durable units than would a competitive industry, as long as the value consumers place on having a working unit in a given period is sufficient high and/or the cost of producing the bulb is sufficiently low. Also, it seems reasonable that one or both of these conditions would have been satisfied in the light bulb case. We are not arguing that we have definitive proof that an important part of the cartel's motivation for reducing durability was the reduction itself, as opposed to the sole motivation being an increase in quality. However, our reading of the evidence leads us to favor the reduction in durability being an important part of the motivation. But more importantly, we believe our theoretical analysis provides a solid theoretical foundation for why the cartel may have found it beneficial to reduce the durability of its bulbs, while alternative theories for reduced durability by a monopolist do not provide such a motivation.

6 CONCLUSION

In many markets the useful lifetime of a product is below what is easily achievable, where this can occur either because built-in durability is low or because of frequent new-product introductions that make used units obsolete. Previous research that has focused on this observation has mostly employed monopoly/market power models to explain the behavior, but in many instances in which the behavior is observed the market seems competitive. In this paper, we have explored how time-inconsistent/present-biased consumer preferences can lead to durability below efficient levels in both competitive and monopoly models. We show that, given both types of market structures, present-biased consumer preferences can lead to built-in durability below efficient levels. Also, given perfect competition but not monopoly, present bias can also lead to quicker than efficient new-product introductions that make used units obsolete. One factor that leads to these results is that a present-biased consumer has an incentive to move consumption from the future to the present more than is efficient from a long-run perspective, and this leads consumers to have a reduced willingness to pay the incremental cost associated with higher built-in durability.

There are a number of directions in which the analysis in this paper could be extended. One direction we feel is of particular interest is enriching the analysis of new-product introductions to allow for various important real-world factors such as heterogeneous consumers, secondhand-market trade, and R&D investments. Moving in these directions would introduce issues of price discrimination, time inconsistency, and adverse selection as found in various papers such as Waldman (1993), Fudenberg and Tirole (1998), and Hendel and Lizzeri (1999b). We believe it would be of interest to investigate how the possibility of frequent new-product introductions due to present bias interacts with complications that arise when factors such as secondhand-market trade and R&D investments are incorporated into the analysis.

Another direction we believe is of interest is to investigate renting or leasing when consumers have present bias. In this paper we assume that consumers purchase the durable good which is a reasonable assumption for many durable goods markets such as the market for light bulbs discussed in Section V. But in many other durable goods markets renting or leasing is common. There is an extensive literature that explores various roles that renting or leasing can play in durable goods markets such as avoiding time inconsistency, reducing adverse selection, and creating moral hazard problems (see, for example, Bulow (1982), Henderson and Ioannides (1983), and Johnson and Waldman (2003)). We feel that in some cases leasing or renting may play an important role in responding to incentives created by present bias. We thus believe that systematically investigating interactions between present bias and renting/leasing would be an interesting direction for future research.

Finally, in our analysis we have abstracted away from borrowing and saving. Since borrowing and purchasing cheaper goods of limited durability are alternative ways that consumers can move consumption from the future to the present, the two behaviors should be substitutes in settings characterized by present-biased consumers. Investigating this substitutability is an additional topic for future research.

7 REFERENCES

Acharya, Subas, David Jimenex-Gomez, Dmitri Rachinskii, and Alejandre Rivera. 2020. "Present Bias and the Value of Sophistication." Mimeo, University of Texas at Dallas, Mathematics Department.

Anderson, Simon and Victor Ginsburgh. 1994. "Price Discrimination via Second-Hand Markets." European Economic Review, 38(1): 23-44.

Bar-Gill, Oren and Andrew Hayashi. 2021. "Present Bias and Debt-Financed Durable Goods." Discussion Paper No. 1055, Harvard Law School.

Brox, Jane. 2010. Brilliant: The Evolution of Artificial Light. Houghton Mifflin Harcourt, New York.

Bulow, Jeremy. 1982. "Durable-Goods Monopolists." Journal of Political Economy, 90(2): 314-332.

Bulow, Jeremy. 1986. "An Economic Theory of Planned Obsolescence." Quarterly Journal of Economics, 101(4): 729-749.

Choi, Jay Pil. 1994. "Network Externality, Compatibility Choice, and Planned Obsolescence." Journal of Industrial Economics, 42(2): 167-182.

Coase, Ronald H. 1972. "Durability and Monopoly." Journal of Law and Economics, 15(1): 143-149.

Fudenberg, Drew and Jean Tirole. 1998. "Upgrades, Trade-ins, and Buy-backs." Rand Journal of Economics, 29(2): 235-258.

Grout, Paul A. and In-Uck Park. 2005. "Competitive Planned Obsolescence." Rand Journal of Economics, 36(3): 596-612.

Hendel, Igal and Alessandro Lizzeri. 1999a. "Interfering with Secondary Markets." Rand Journal of Economics, 30(1): 1-21.

Hendel, Igal and Alessandro Lizzeri. 1999b. "Adverse Selection in Durable Goods Markets." American Economic Review, 89(5): 1097-1115. Henderson, J.V. and Y.M. Ioannides. 1983. "A Model of Housing Tenure Choice." American Economic Review, 73(1): 98-113.

Hutton Power & Light. 2019. "LED vs. Incandescent Lighting: A Cost Comparison." Blog Post.

Johnson, Justin P. and Michael Waldman. 2003. "Leasing, Lemons, and Buybacks." Rand Journal of Economics, 34(2): 247-265.

Kang, Jingoo and Minwook Kang. 2022. "Durable Goods as Commitment Devices Under Quasi-Hyperbolic Discounting." Journal of Mathematical Economics, 99: Article 102561.

Kleiman, E. and T. Ophir. 1966. "The Durability of Durable Goods." Review of Economic Studies, 33(2): 165-178.

Krajewski, Markus. 2014. "The Great Lightbulb Conspiracy." IEEE Spectrum, 51(10): 56-61.

Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting." Quarterly Journal of Economics, 112(2): 443-477.

Levhari, David and T. Srinivasan. 1969. "Durability of Consumption Goods: Competition versus Monopoly." American Economic Review, 59(1): 102-107.

Loewenstein, George and Drazen Prelec. 1992. "Anomolies in Intertemporal Choice: Evidence and an Interpretation." Quarterly Journal of Economics, 107(2): 573-597.

Meier, Stephan and Charles Sprenger. 2010. "Present-Biased Preferences and Credit Card Borrowing." American Economic Journal: Applied Economics, 2(1): 193-210.

Mussa, Michael and Sherwin Rosen. 1978. "Monopoly and Product Quality." Journal of Economic Theory, 18(2): 301-317.

Nocke, Volker and Martin Peitz, 2003. "Hyperbolic Discounting and Secondary Markets." Games and Economic Behavior, 44(1): 77-97.

O'Donoghue, Ted and Matthew Rabin. 1999. "Doing it Now or Later." American Economic Review, 89(1): 103-124.

O'Donoghue, Ted and Matthew Rabin. 2002. "Addiction and Present-Biased Preferences." Working Paper No. E02-312, UC Berkeley, Department of Economics.

O'Donoghue, Ted and Matthew Rabin. 2015. "Present Bias: Lessons Learned and to be Learned." American Economic Review, 105(5): 273-279.

Pesendorfer, Wolfgang. 1995. "Design Innovation and Fashion Cycles." American Economic Review, 85(4): 771-792.

Schmalensee, Richard. 1970. "Regulation and the Durability of Goods." Bell Journal of Economics and Management Science, 1(1): 54-64.

Schmalensee, Richard. 1979. "Market Structure, Durability, and Quality: A Selective Survey." Economic Inquiry, 17(2): 177-196.

Sieper, E. and Peter Swan. 1973. "Monopoly and Competition in the Market for Durable Goods." Review of Economic Studies, 40(3): 333-351.

Strotz, R.H. 1955. "Myopia and Inconsistency in Dynamic Utility Maximization." Review of Economic Studies, 23(3): 165-180.

Swan, Peter L. 1970. "Durability of Consumption Goods." American Economic Review, 60(5): 884-894.

Swan, Peter L. 1971. "The Durability of Goods and Regulation of Monopoly." Bell Journal of Economics and Management Science, 2(1): 347-357.

Tabuchi, Hiroko. 2022. "New Rules Will End the Century-Long Run of Classic Light Bulbs." New York Times, April 26, 2022.

The Monopolies and Restrictive Practices Commission. 1951. Report on the Supply of Electric Lamps. Her Majesty's Stationary Office, London.

Waldman, Michael. 1993. "A New Perspective on Planned Obsolescence." Quarterly Journal of Economics, 108(1): 273-283.

Waldman, Michael. 1996a. “Durable Goods Pricing When Quality Matters.” Journal of Business, 69(4): 489-510.

Waldman, Michael. 1996b. “Planned Obsolescence and the R&D Decision.” Rand Journal of Economics, 27(3): 583-595.

Waldman, Michael. 2003. “Durable Goods Theory for Real World Markets.” Journal of Economic Perspectives, 17(1): 131-154.

8 APPENDIX

8.1 Solve the First Best

Here, I solve the model of linear utility $f(x_{i,t}) = x_{i,t}$ in the numeraire goods. Let the utility function on the numeraire goods be $u_q(w) = w$. Which means the marginal utility and the price of the numeraire goods are both normalized to 1.

Assume $c(\theta)$ is strictly increasing and convex in θ. Let $c'(0) = 0$ and $c'(1) = \infty$. This is the Inada condition to ensure the interior solution for time consistent consumers in competitive market. Also, assume there is at least one θ_0 such that $c(\theta_0) \leq \frac{v}{1-\delta\theta_e}$. This means there are at least some durability levels such that the consumer surplus of the light bulb surplus is larger than the cost.

The first best is achieved by maximizing the total surplus (TS), by choosing the optimal durability

$$TS = \max_{\theta} \frac{v+w}{1-\delta} - \left(c(\theta) + \frac{(1-\theta)\delta c(\theta)}{1-\delta} \right) \tag{9}$$

The first term is the total surplus from wage and the discounted utility from the durable goods. The second term is the total discounted cost of supplying the durable goods. The consumer always buy a durable good when it breaks. Every period, there is a probability $1-\theta$ the light bulb breaks and a new one needs to be produced. So the expected stream of production cost is $c(\theta) + \delta(1-\theta)c(\theta) + \delta^2(1-\theta)c(\theta)$... which equals to $c(\theta) + \frac{\theta(1-\delta)c(\theta)}{1-\delta}$

Thus, the corresponding FOC is:

$$\textit{[FOC] First Best:} \;\; 0 = -c'(\theta) + \frac{\delta c(\theta)}{1-\theta\delta} \tag{10}$$

Denote the θ that satisfies the first best as θ^*

8.2 Proof of Proposition 1:

Here, we want to solve the durability choice for time consistent consumers in competitive and monopoly market.

Competitive Market: We first establish that the competitive market price of a good with durability θ has price equal to $c(\theta)$. Since the marginal cost is independent of the quantity sold, the competitive price should not depend on the quantity sold. Also, the zero profit condition in competitive market guarantees that if a firm sets a price higher than $c(\theta)$, it will not sell at all. If a firm setts a price lower than $c(\theta)$, then the firm loses money. So, it must be that the market price equals to $c(\theta)$.

Let $v_1(L = 0)$ be the value of for a consumer without a working light bulb at the beginning of a period. Let $v_1(\theta, L = 1)$ be the value that a consumer start with a durable good with durability

θ. Then, when the consumers choose the optimal durability level, the value functions should take the following forms:

$$\begin{aligned} v_1(L=0) &= \max_\theta v + w - c(\theta) + \delta\theta v(\theta, L=1) + \delta(1-\theta)v(v_1, L=0) \\ v_1(\theta, L=1) &= v + w + \delta\theta v_1(\theta, L=1) + \delta(1-\theta)v_1(L=0) \end{aligned} \tag{11}$$

$$\begin{aligned} v_1(L=0) &= \max_{\theta_1^*} \frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* w + \frac{1-\delta\theta_1^*}{1-\delta}(w - c(\theta_1^*)) \\ &= \max_{\theta_1^*} \frac{v+w}{1-\delta} - \frac{1-\delta\theta_1^*}{1-\delta}c(\theta_1^*) \\ v_1(\theta_1^*, L=1) &= \frac{v}{1-\delta} + (\frac{\delta\theta_1^*}{1-\delta} + 1)w + \frac{\delta - \delta\theta_1^*}{1-\delta}(w - c(\theta_1^*)) \\ &= \frac{v+w}{1-\delta} - \frac{\delta(1-\theta)}{1-\delta}c(\theta) \end{aligned} \tag{12}$$

Taking the derivative with respect to the only choice variable θ_1^*, the corresponding FOC becomes:

$$\text{[FOC] } \theta_1: \ \ \delta c(\theta_{1,C}^*) - (1-\delta\theta_{1,C}^*)c'(\theta_{1,C}^*) = 0 \tag{13}$$

The FOC of the competitive market with time consistent consumers coincides the first best. Thus, let $\theta_{1,C}^*$ denotes the choice of durability of the competitive market when the consumers are time consistent. This is established as a benchmark for later sections.

Monopoly Market: To simplify the monopolist problem, assume in addition that $c(\theta) \leq \frac{v}{1-\delta\theta^*}$, so the firm earns positive profit on all durability levels.

For the time consistent consumers, the firm choose θ and price P to maximize the following profit:

$$\pi(\theta) = \max_\theta (P - c(\theta)) + \delta(1-\theta)(P - c(\theta)) + \delta^2(1-\theta)(P - c(\theta)) + ... \tag{14}$$

Because every period, there is a fraction $1-\theta$ of the consumers' durable goods break and has to pay P to get a new one.

Here P is the maximum willingness to pay for the durable goods. So, the consumer is indifferent between reduce the contemporary purchase of numeraire goods by P, and derive the utility V form the durable goods every period. However, the durable goods only last to the next period with probability θ. So, the stream of payoff that can be generated from the payoff is

$$\begin{aligned} p &= v + \delta\theta v + \delta^2\theta^2 v + ... \\ &= \frac{v}{1-\delta\theta} \end{aligned} \tag{15}$$

It is the same as deriving the P from the consumer's indifference condition. Then, we can plug the prince into the profit function as shown below.

$$
\begin{aligned}
\pi(\theta) &= \max_{\theta} \frac{(1-\theta)\delta(\frac{v}{1-\delta\theta} - c(\theta))}{1-\delta} + \frac{v}{1-\delta\theta} - c(\theta) \\
&= \max_{\theta}(1 + \frac{(1-\theta)\delta}{1-\delta})(\frac{v}{1-\delta\theta} - c(\theta))
\end{aligned} \tag{16}
$$

The FOC is the same as before:

$$
\begin{aligned}
&\textit{FOC: } -\frac{\delta}{1-\delta}(\frac{v}{1-\delta\theta} - c(\theta)) + (1 + \frac{(1-\theta)\delta}{1-\delta})(\frac{\delta v}{(1-\delta\theta)^2} - c'(\theta)) = 0 \\
&\textit{FOC: } -\delta(\frac{v}{1-\delta\theta} - c(\theta)) + (1-\delta\theta)(\frac{\delta v}{(1-\delta\theta)^2} - c'(\theta)) = 0 \\
&\textit{FOC: } \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0
\end{aligned} \tag{17}
$$

Thus, when the consumer is time consistent, the monopolist chooses exactly the same durability level as the first best.

8.3 Proof of Proposition 2:

Here, we solve the competitive/monopoly market durability for present biased consumers.

Competitive Market In the prove of proposition 1, we derived the value functions for the time consistent consumers. These value functions are the same as the expected value for present based consumers, so we define the following terms:

$$
\begin{aligned}
v_1(L=0) &= \frac{v + w - (1-\delta\theta^*)c(\theta^*)}{1-\delta} \\
v_1(\theta^*, L=1) &= \frac{v}{1-\delta} + (\frac{\delta\theta^*}{1-\delta} + 1)w + \frac{\delta - \delta\theta^*}{1-\delta}(w - c(\theta^*)) \\
&= \frac{v + w(1-\delta+\delta\theta^*+\delta-\delta\theta^*) + (\delta-\theta^*)c(\theta^*)}{1-\delta} \\
&= \frac{v + w + \delta(1-\theta^*)c(\theta^*)}{1-\delta} \\
v_1(\theta, L=1) &= v + w + \delta\theta v_1(\theta, L=1) + \delta(1-\theta)v_1(\theta^*, L=1) \\
&= \frac{v+w}{1-\delta\theta} + \frac{\delta(1-\theta)}{1-\delta\theta}v_1(\theta^*, L=1)
\end{aligned} \tag{18}
$$

$v_1(L=0)$ is the value when the consumer does not own a durable goods. $v_1(\theta^*, L=1)$ is the value of the consumer has a first best durable goods. $v_1(\theta, L=1)$ is the value of the consumer having a goods of durability θ which can be different from θ^*. Notice that the second term $(v_1(\theta^*, L=1))$ is a constant and only depends on the value of parameters. The third term ($v_1(\theta, L=1)$) is important for the analysis of the present biased part of the game, as I will show below.

After solving for the time consistent part, we can look at the present biased part of the value function. This is the period zero problem. The consumer always expects himself to choose the durability θ^* in the future, but not on the current period. Let the present choice of durability be $\theta^*_{P,N'}$. The value function and maximization problems are the following:

$$\begin{aligned}
v(L=0) &= \max_{\theta}\ w + v - c(\theta) + \beta\delta\theta v_1(\theta, L=1) + \beta\delta(1-\theta)v_1(L=0) \\
&= w + v - c(\theta) + \beta\delta\theta(\frac{v+w}{1-\delta\theta} + \frac{\delta(1-\theta)}{1-\delta\theta}v_1(\theta^*, L=1)) + \beta\delta(1-\theta)v_1(L=0) \\
&= -c(\theta) + (w+v)(1+\frac{\beta\delta\theta}{1-\delta\theta}) + \beta\frac{(1-\theta)\delta}{1-\theta\delta}v_1(L=0) \\
&= w + v - c(\theta) + \beta[(v+w)\frac{-\theta\delta^2+\delta}{(1-\delta)(1-\delta\theta)} - \frac{(1-\theta)\delta(1-\delta\theta^*)}{(1-\delta\theta)(1-\delta)}c(\theta^*)] \\
&= w + v - c(\theta) + \beta[(v+w)\frac{\delta}{1-\delta} - \frac{(1-\theta)\delta(1-\delta\theta^*)}{(1-\delta\theta)(1-\delta)}c(\theta^*)]
\end{aligned} \tag{19}$$

The corresponding first order conditions are:

$$\begin{aligned}
\textit{[FOC] }\theta\textit{:}\quad \frac{1}{\beta}c'(\theta^*) &= \delta[v_1(\theta^*, L=1) + \frac{\partial v_1(\theta^*, L=1)}{\partial\theta^*} - v_1(L=0)] \\
0 &= -c'(\theta) + \beta[\frac{1-\delta}{(1-\delta\theta)^2}\frac{\delta(1-\delta\theta^*)c(\theta^*)}{1-\delta}] \\
0 &= -c'(\theta) + \beta[\frac{\delta(1-\delta\theta^*)c(\theta^*)}{(1-\delta\theta)^2}] \\
\textit{[FOC] }\theta^*\textit{:}\quad \delta c(\theta^*) &= (1-\delta\theta^*)c'(\theta^*)
\end{aligned} \tag{20}$$

Combine the above FOCs', we can clearly derive the following relation:

$$(1-\delta\theta_{P,N'})^2 c'(\theta_{P,N'}) = \beta(1-\delta\theta_1^*)^2 c'(\theta_1^*) \tag{21}$$

Since $c'(\theta)$ is strictly increasing with θ, and $c'(0)=0$, we can conclude $\theta_{P,N'} < \theta^*$. So present bias consumers must purchase less durable goods than non-present bias ones.

Monopoly Market We solve the monopoly price for the preset-bias naive consumer. The problem is easier with linear utility function, because the consumer anticipates that she will not buy the product in the future. So, we can solve for the willingness to pay for the light bulb buy the naive consumer as the price charged by the monopolist:

$$p_{\beta}(\theta) = v + \beta\frac{\delta\theta v}{1-\delta\theta} \tag{22}$$

The firm is still time consistent, so we have the following profit:

$$\begin{aligned} \pi(\theta) &= \max_{\theta} p_{\beta}(\theta) - c(\theta) + \frac{(1-\theta)\delta(p_{\beta}(\theta) - c(\theta))}{1-\delta} \\ &= \max_{\theta}(p_{\beta}(\theta) - c(\theta))(1 + \frac{(1-\theta)\delta}{1-\delta}) \\ &= \max_{\theta}(v + \beta\frac{\delta\theta v}{1-\delta\theta} - c(\theta))(1 + \frac{(1-\theta)\delta}{1-\delta}) \end{aligned} \tag{23}$$

This is very similar to the profit from time consistent consumers. The only difference is the price faced by the naive consumer should be lower. Similarly, we can derive the FOC:

$$\begin{aligned} &FOC\text{:} -\frac{\delta}{1-\delta}(v + \beta\frac{\delta\theta v}{1-\delta\theta} - c(\theta)) + (1 + \frac{(1-\theta)\delta}{1-\delta})(\frac{v\delta\beta}{(1-\delta\theta)^2} - c'(\theta)) = 0 \\ &FOC\text{:} -\delta(v + \beta\frac{\delta\theta v}{1-\delta\theta} - c(\theta)) + (1-\delta\theta)(\frac{v\delta\beta}{(1-\delta\theta)^2} - c'(\theta)) = 0 \\ &FOC\text{:} -\delta(v + \beta\frac{\delta\theta v}{1-\delta\theta}) + \frac{v\delta\beta}{(1-\delta\theta)} + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} -\delta(\frac{v - v\delta\theta + \beta\delta\theta v}{1-\delta\theta}) + \frac{v\delta\beta}{(1-\delta\theta)} + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} -(\frac{\delta v - v\delta^2\theta + \beta\delta^2\theta v}{1-\delta\theta}) + \frac{v\delta\beta}{(1-\delta\theta)} + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} (\frac{+v\delta^2\theta(1-\beta)}{1-\delta\theta}) + \frac{v\delta(\beta-1)}{(1-\delta\theta)} + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} (1-\beta)(\frac{-v\delta + v\delta^2\theta}{1-\delta\theta}) + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} (1-\beta)(\frac{v\delta(-1+\delta\theta)}{1-\delta\theta}) + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \\ &FOC\text{:} -(1-\beta)v\delta + \delta c(\theta) - (1-\delta\theta)c'(\theta) = 0 \end{aligned} \tag{24}$$

To compare the durability choice of the time consistent and present bias consumers, we can directly look at the FOCs. Let θ^* be the monopoly durability for time consistent consumer (we showed it is the first best durability). If we plug it into the FOC for the present biased consumer, FOC for the present bias consumer must be negative because the term $\frac{-v\delta+v\delta^2\theta}{1-\delta\theta}$) must be negative. Which means the monopolist chooses strictly less durable goods for the present bias consumers.

Proof for point (i): When consumers are present bias, their willingness to pay decrease, which directly causes the profit for the monopolist to be lower than the time consistent consumer market.

With the lower price, time consistent is always willing to pay for the product at the monopoly price for the present bias consumers. So, the consumer shares a larger surplus compare to the time consistent case. Thus, the consumer surplus may increase. However, when the durability is distorted, it is also more costly to produce the light bulbs, which can decrease the consumer surplus. When we sum up the two effects on the consumer, the total consumer welfare change is uncertain.

8.4 Proof of Corollary 1:

In the previous sections, we have derived the FOCs of the present biased and time consistent consumers in competitive and monopolist markets. Then, we can do the comparative statics using the FOCs. Assume the cost function $c(\theta)$ is strictly increasing for $\theta \in [0,1]$ and strictly convex. Assume $c'(0) = 0$ and $c'(1) = \infty$, to rule out the corner solutions.

$$\begin{aligned} &\textit{[FOC] First Best:}\ \ 0 = -c'(\theta^*) + \frac{\delta c(\theta^*)}{1-\theta^*\delta} \\ &\textit{[FOC] Competitive:}\ \ 0 = -c'(\theta) + \beta[\frac{\delta(1-\delta\theta^*)c(\theta^*)}{(1-\delta\theta)^2}] \\ &\textit{[FOC] Monopolist:}\ \ 0 = -c'(\theta) + \frac{\delta c(\theta)}{(1-\delta\theta)} - (1-\beta)v\delta \end{aligned} \tag{25}$$

Notice that when $\beta = 1$, all three conditions coincide. It means both the competitive market and the monopolist market results in the first best durability.

First, in the competitive market, the present bias consumers always choose a less durable good than time consistent consumer. This can be easily derived from comparing ([FOC] Competitive) for the case of $\beta = 1$ and $\beta < 1$.

Second, the monopolist may choose a less durable goods for the present bias consumer compare to the time consistent consumer.

Third, we compare the durability choice in competitive and monopolistic market for consumers when $\beta \leq 1$. Let $\theta_{P,N'}$ be the durability of goods in the competitive market, and let $\theta_{M,N'}$ be the durability of goods in the monopolist market. We compare the size of them by plug in $\theta_{P,N'}$ to the FOC of the monopolist. And if the resulting number is positive, it means the monopolist can still increase profit by increasing the durability if the durability is currently $\theta_{P,N'}$. Thus, $\theta_{M,N'} > \theta_{P,N'}$. Otherwise the monopolist wants to decrease the durability and thus, $\theta_{M,N'} > \theta_{P,N'}$. By plug $\theta_{P,N'}$ into the FOC for the monopolist, the right hand side of the FOC for the monopolist becomes:

$$\begin{aligned} &-\frac{\beta\delta(1-\delta\theta_1^*)c(\theta_1^*)}{(1-\delta\theta_{P,N'})^2} + \frac{\delta c(\theta_{P,N'})}{(1-\delta\theta_{P,N'})} - (1-\beta)v\delta \\ &= \frac{\delta c(\theta_{P,N'})(1-\delta\theta_{P,N'}) - \beta\delta(1-\delta\theta^*)c(\theta^*)}{(1-\delta\theta_{P,N'})^2} - (1-\beta)v\delta \end{aligned} \tag{26}$$

We can show that this can be positive or negative depending on all kinds of parameters. It is very hard to compare the two FOCs in detail. But, we can discuss some extreme cases.

For instance, when the cost is very low so that $c(\theta)$ is close to zero. Then the sign of function (26) will be mainly determined by the last term $-(1-\beta)v\delta$, which is negative. Then, we will have planned obsolescence as the monopolist has the incentive to set a lower durability then $\theta_{P,N'}$. Similarly, if v is large, it also dominates the cost and the FOC is also negative. So, we have planned obsolescence too.

On the other hand, if the cost is large and $\theta_{P,N'} < \theta^*$. Then it is possible that the first term of the function 26 is large and causing the entire thing to be positive. Same thing happens when v is small.

For another example when $\beta = 0$, then in the competitive equilibrium, the consumer choose zero durability. However, for the monopolist, if we plug $\theta = 0$ into its FOC, we realize that it is negative because when comparing $\frac{(1-\delta\theta^*)c(\theta^*)}{1-\delta}$ and $\frac{v}{1-\delta}$. The former one is the cost of supplying the optimal bundle till infinity, while the latter is the value of having the light bulb till infinity. By assumption of the cost, the latter one should be larger. So, the sum of the first two terms are negative. It means the monopolist have the incentive to further reduce the durability.

8.5 Proof of Proposition 3:

In this section, I solve for the stationary equilibrium for the sophisticated consumers.

Competitive Market Since every time the light bulb breaks, the consumer faces the exact same problem, and thus should correctly anticipate her choice of durability in the future to always be the same durability for whenever the light bulb breaks. Let this anticipated level of durability be θ_1. Then the current period self best respond to the anticipation and choose the current period θ. Finally, the Stationary Markov perfect equilibrium requires fixed point constraint that $\theta = \theta_1$. So, we can solve for the both durability.

We first define a new notation $v_1(\theta_1, L=0)$ be the expected continuation payoff if the consumer anticipates the future selves always choose the durability θ_1. Let $v_1(\theta_1, L=1)$ be the expected time consistent value of having a working light bulb with durability θ_1

$$
\begin{aligned}
v_1(\theta_1, L=0) &= v + w - c(\theta_1) + \delta\theta_1 v_1(\theta_1, L=1) + \delta(1-\theta_1)v_1(\theta_1, L=0) \\
v_1(\theta_1, L=1) &= v + w + \delta\theta_1 v_1(\theta_1, L=1) + (1-\theta_1)v_1(\theta_1, L=0)
\end{aligned} \tag{27}
$$

Solving the system of equations, we can then get the following expressions:

$$
\begin{aligned}
v_1(\theta_1, L=0) &= \frac{v+w}{1-\delta} - \frac{(1-\delta\theta_1)}{1-\delta}c(\theta_1) \\
v_1(\theta_1, L=1) &= \frac{v+w}{1-\delta} - \frac{\delta(1-\theta_1)}{1-\delta}c(\theta_1)
\end{aligned} \tag{28}
$$

Let $v_1(\theta, \theta_1, L=1)$ be the continuation utility of a future self who carries a light bulb with durability θ, while the current self also anticipates that if the bulb breaks, the future self will replace it with a light bulb with durability θ_1.

$$
\begin{aligned}
v_1(\theta,\theta_1,L=1) &= w+v+\delta\theta v_1(\theta,\theta_1,L=1)+\delta(1-\theta)v_1(\theta_1,L=0) \\
v_1(\theta,\theta_1,L=1) &= \frac{w+v}{1-\delta\theta}+\frac{\delta(1-\theta)v_1(\theta_1,L=0)}{1-\delta\theta}
\end{aligned} \tag{29}
$$

Let $v(L=0)$ denotes the value function when the current period consumer does not have a working light bulb to start with. Thus, we have the following relation:

(30)

$$
\begin{aligned}
v(L=0) &= \max_{\theta}\ w-c(\theta)+v+\beta\delta\theta v_1(\theta,\theta_1,L=1)+\beta\delta(1-\theta)v_1(\theta_1,L=0) \\
&= \max_{\theta}\ w-c(\theta)+v+\beta\delta\theta\frac{w+v}{1-\delta\theta}+\beta\delta\theta\frac{\delta(1-\theta)v_1(\theta_1,L=0)}{1-\delta\theta}+\beta\delta(1-\theta)v_1(\theta_1,L=0) \\
&= \max_{\theta}\ w+v+\beta\delta\theta\frac{w+v}{1-\delta\theta}-c(\theta)+v_1(\theta_1,L=0)\beta\delta(\frac{\theta(1-\theta)\delta}{1-\delta\theta}+\frac{(1-\theta)(1-\delta\theta)}{1-\delta\theta}) \\
&= \max_{\theta}\ w+v+\beta\delta\frac{\theta(w+v)}{1-\delta\theta}-c(\theta)+\beta\delta\frac{1-\theta}{1-\delta\theta}v_1(\theta_1,L=0)
\end{aligned}
$$

Thus, for the current self, the FOC becomes the following one:

$$
\begin{aligned}
&\textit{[FOC] }\theta\text{:}\quad -c'(\theta)+\beta\delta\frac{w+v}{(1-\delta\theta)^2}+\beta\delta\frac{\delta-1}{(1-\delta\theta)^2}v_1(\theta_1,L=0)=0 \\
&\textit{[FOC] }\theta\text{:}\quad -c'(\theta)+\beta\delta\frac{w+v}{(1-\delta\theta)^2}+\beta\delta\frac{\delta-1}{(1-\delta\theta)^2}[\frac{v+w}{1-\delta}-\frac{(1-\delta\theta_1)}{1-\delta}c(\theta_1)]=0 \\
&\textit{[FOC] }\theta\text{:}\quad -c'(\theta)+\beta\delta\frac{(1-\delta\theta_1)}{(1-\delta\theta)^2}c(\theta_1)=0
\end{aligned} \tag{31}
$$

This FOC is very similar to the FOC for the naive consumer. If $\theta_1=\theta_1^*$ which means the sophisticated consumer expects himself to buy the light bulb with the most efficient durability, then the current period decision of the sophisticated consumer will be exactly the same as the naive consumer. However, this solution violates the stationarity assumption, because the previous solution is true, then the sophisticated consumer does not correctly anticipate the future self's decision.

Let $\theta_{P,S'}$ ($\theta_{P,N'}$) be the solution to the sophisticated (naive) consumer's current period problem in competitive market. We can rewrite the FOCs and get the following expression.

$$
\begin{aligned}
&\textit{[FOC] Sophi:}\ \ 0 = -c'(\theta_{P,S'}) + \beta\delta\frac{(1-\delta\theta_{P,S'})}{(1-\delta\theta_{P,S'})^2}c(\theta_{P,S'}) \\
&\textit{[FOC] Naive:}\ \ 0 = -c'(\theta_{P,N'}) + \beta\delta\frac{(1-\delta\theta^*)}{(1-\delta\theta_{P,N'})^2}c(\theta^*) \qquad (32)\\
&\textit{[FOC] First Best:}\ \ 0 = -c'(\theta^*) + \frac{\delta}{1-\theta^*\delta}c(\theta^*)
\end{aligned}
$$

It is unclear how to do the comparative statics without a functional form of the cost function $c(\theta)$. The problem is that $c'(\theta_{P,S'})$ and $c(\theta_{P,S'})$ both decrease with $\theta_{P,S'}$. So, it is not clear when does the two terms match. The detailed shape of the cost function is really important for the solution.

To do the comparative statics we need to put further assumption on the cost function $c(\theta)$. The most important thing is to compare the cost with the benchmark $c_{indiff}(\theta) = \frac{c_0}{1-\delta\theta}$

Assumption 1. *Let θ^* be the first best durability. For all $\theta < \theta^*$, the cost function is such that $c(\theta)(1-\delta\theta)$ is strictly decreasing. For all $\theta \geq \theta^*$, the cost function is such that $c(\theta)(1-\delta\theta)$ is strictly increasing.*

The above assumption is equivalent to assuming a unique solution to the first best durability. The following is a graphical illustration of the cost function.

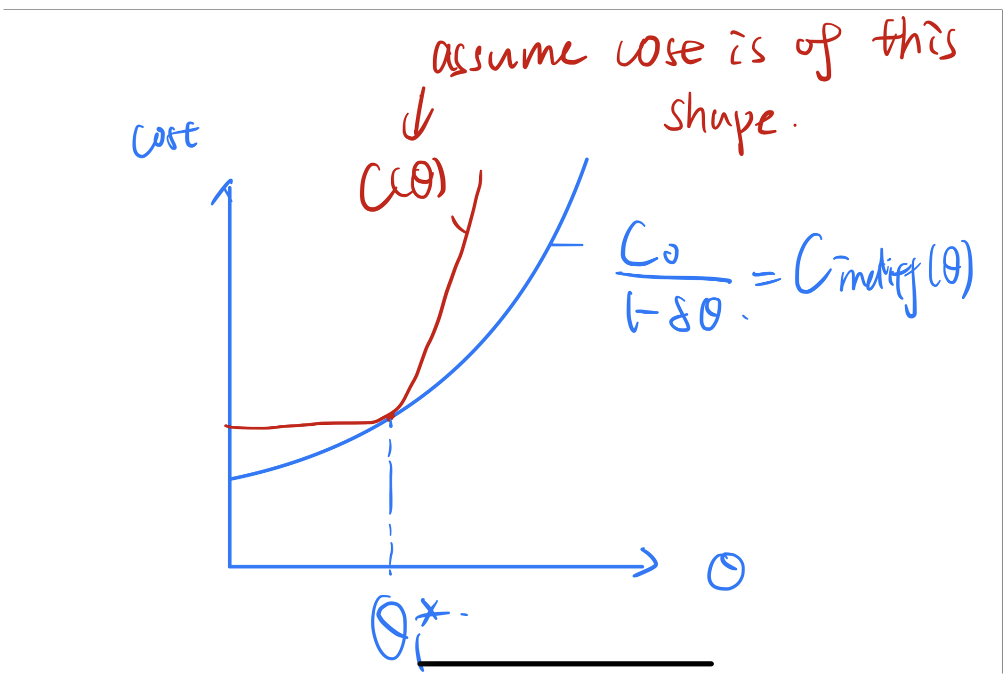

Figure 1: Red line is the cost function

Then we can compare the durability of the naive consumer and the sophisticated consumer by plug in $\theta_{P,N'}$ to the FOC of the sophisticated consumer. The right hand side of FOC becomes the following

$$\text{[FOC] RHS:} \quad -c'(\theta_{P,N'})(1-\delta\theta_{P,N'})^2 + \beta\delta c(\theta_{P,N'})(1-\delta\theta_{P,N'}) \tag{33}$$

Using the FOC of the naive consumer, we know that

$$-c'(\theta_{P,N'})(1-\delta\theta_{P,N'})^2 = \beta\delta(1-\delta\theta^*)c(\theta^*)$$

So making the above substitution, the FOC of the sophisticated consumer further becomes:

$$\text{[FOC] RHS:} \quad -\beta\delta(1-\delta\theta^*)c(\theta^*) + \beta\delta c(\theta_{P,N'})(1-\delta\theta_{P,N'}) \tag{34}$$

We know that $\theta_{P,N'} < \theta^*$, so the right hand side of the FOC for the sophisticated consumer as to be positive according to assumption 1. It means $\theta_{P,S'} > \theta_{P,N'}$.

Similarly, we can plug θ^* to the right hand side of the FOC for the sophisticated consumer.

$$\text{[FOC] RHS:} \quad -c'(\theta^*)(1-\delta\theta^*)^2 + \beta\delta c(\theta^*)(1-\delta\theta^*) \tag{35}$$

By the FOC of the first best, we know that

$$-c'(\theta^*)(1-\delta\theta^*)^2 = \delta c(\theta^*)(1-\delta\theta^*)$$

Thus, we know the value of the right hand side of FOC sophisticated consumer is the following

$$\begin{aligned} \text{[FOC] RHS:} \quad & -\delta c(\theta^*)(1-\delta\theta^*) + \beta\delta c(\theta^*)(1-\delta\theta^*) \\ & = (\beta-1)c(\theta^*)(1-\delta\theta^*) < 0 \end{aligned} \tag{36}$$

Thus, we can conclude $\theta_{P,N'} < \theta_{P,S'} < \theta^*$.

Monopoly Market In the monopoly market, the sophisticated consumer behaves exactly the same as naive consumers, because both anticipates to buy the product with the price and durability chosen by the monopolist. In other words, both the naive and sophisticated have the same anticipation of the continuation payoff given the firm's choice. Thus, the monopolist will also chooses the same durability for both the naive and sophisticated consumers. So, $\theta_{M,S'} = \theta_{M,N'} < \theta^*$ for all $\beta < 1$

8.6 Concave Utility Function

8.6.1 First Best

Assume that the cost of production is low such that there exists some durability level that the cost of producing the durable goods is smaller than the benefits that it generates.

We first look at the social planner problem to maximize the total welfare. Let $T(\theta)$ stands for total welfare when the durability is θ.

$$T(\theta) = \max_{\theta} \frac{v}{1-\delta} + u(w - c(\theta)) + \frac{(1-\theta)\delta u(w - c(\theta))}{1-\delta} + \frac{\theta \delta u(w)}{1-\delta} \tag{37}$$

As we will see later, this maximization problem is exactly the same as the maximization problem for time consistent consumer. So the First Best is achieved by the competitive market.

$$\textit{[FOC]}\ \theta_{FB}\text{:}\ \ \frac{\delta}{1-\delta} u_q(w) - \frac{\delta}{1-\delta} u_q(w - c(\theta_1^*)) - \frac{1-\delta\theta_1^*}{1-\delta} u_q'(w - c(\theta_1^*)) c'(\theta_1^*) = 0 \tag{38}$$

8.6.2 Competitive market, infinite periods, no saving and borrowing

The infinite period problem will be much simper to solve, if saving and borrowing are not needed.

Let $v(\theta, L = 1)$ be the utility of having a light bulb with durability θ at the beginning of the period. $v(L = 0)$ means the utility when the does not have a light bulb. Let $v_1(\theta, L = 1)$ denotes the anticipated value for the future period of owning a light bulb with durability θ. $v_1(L = 0)$ is the anticipated value for the future period of not having a light bulb. $v_1(\cdot)$ is not β discounted.

Notice that there is no maximization operator for $v(\theta, L = 1)$ and $v_1(L = 0)$. Because whenever the consumer has a working light bulb, she does not have a choice variable on that period and just spend all her income on the consumption goods.

$$\begin{aligned} v(L = 0) &= \max_{\theta} u_q(m - c(\theta)) + v + \beta\delta\theta v_1(\theta, L = 1) + \beta\delta(1-\theta) v_1(L = 0) \\ v(\theta, L = 1) &= u_q(m) + v + \beta\delta\theta v_1(\theta, L = 1) + \beta\delta(1-\theta) v_1(L = 0) \end{aligned} \tag{39}$$

We first need to solve the time consistent part of the value function $v_1(\cdot)$It follows the Bellman equation below:

$$\begin{aligned} v_1(L = 0) &= \max_{\theta_1} u_q(w - c(\theta_1)) + v + \delta\theta_1 v_1(\theta_1, L = 1) + \delta(1-\theta_1) v_1(L = 0) \\ v_1(\theta_1, L = 1) &= u_q(w) + v + \delta\theta_1 v_1(\theta_1, L = 1) + \delta(1-\theta_1) v_1(L = 0) \\ &= \frac{1}{1-\delta\theta_1} \left[u_q(m) + v + \delta(1-\theta_1) v_1(L = 0) \right] \end{aligned} \tag{40}$$

Let θ_1^* be the optimal solution. We can then solve the system of equation, and derive the following expression for the value functions:

$$
\begin{aligned}
v_1(L=0) &= \max_{\theta_1^*} \frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*)) \\
v_1(\theta_1^*, L=1) &= \frac{v}{1-\delta} + (\frac{\delta\theta_1^*}{1-\delta} + 1) u_q(w) + \frac{\delta - \delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*))
\end{aligned} \tag{41}
$$

Where θ_1^* is the expected optimal choice of light bulb durability. And it is obtained by solving the following first order condition:

$$
\text{[FOC] } \theta_1: \; \frac{\delta}{1-\delta} u_q(w) - \frac{\delta}{1-\delta} u_q(w - c(\theta_1^*)) - \frac{1-\delta\theta_1^*}{1-\delta} u_q'(w - c(\theta_1^*)) c'(\theta_1^*) = 0 \tag{42}
$$

For future comparative statics, it is easier to keep $v_1(L=0)$ in the value function, so FOC can be written in the following form:

$$
\text{[FOC] } \theta_1: \; 0 = -u_q'(w - c(\theta^*)) c'(\theta^*) + \delta \left[\frac{u_q(w) + v + (1-\theta)\delta v_1(L=0)}{(1-\delta\theta)^2} - \frac{v_1(L=0)}{1-\theta\delta} \right] \tag{43}
$$

Notice that θ_1 is just the expected durability of purchase, however, for all $\beta < 1$, the consumer almost surely never purchase such a light bulb.

We also need to calculate $v_1(\theta, L=1)$ which is the expected future value of owning an light bulb of durability $\theta \neq \theta_1$.

It follows the Bellman equation below:

$$
v_1(\theta, L=1) = u_q(w) + v + \theta\delta v_1(\theta, L=1) + (1-\theta)\delta v_1(L=0) \tag{44}
$$

So we can solve for the $v_1(\theta, L=1)$ explicitly,

$$
v_1(\theta, L=1) = \frac{1}{1-\theta\delta}[u_q(w) + v + (1-\theta)\delta[\frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*))]] \tag{45}
$$

Then, we have all the elements to solve the first period value functions.

First period explicit value function The following is the initial period (0) value when the consumer does not have a working light bulb in hand.

$$
v(L=0) = \max_{\theta} \; u_q(m - c(\theta)) + v + \beta\delta(\theta v(0, L=1) + (1-\theta) v_1(L=0)) \tag{46}
$$

Since the consumer is naive, and thus expects to buy the optimal durability in the future, $v_1(L=0)$ is thus independent of the initial choice of durability θ, and thus can be regarded as a fixed number. The functional form of $v_1(\theta, L=1)$ is also derived before, and can be directly used here.

(47)

$$\begin{aligned}
v(L=0) &= \max_{\theta}\ u_q(m-c(\theta)) + v + \beta\delta(\theta v(0, L=1) + (1-\theta)v_1(L=0)) \\
&= \max_{\theta}\ u_q(m-c(\theta)) + v \\
&+ \frac{\beta\delta\theta}{1-\theta\delta}[u_q(w) + v + (1-\theta)\delta[\frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta}u_q(w-c(\theta_1^*))]] \\
&+ \beta\delta(1-\theta)[\frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta}u_q(w-c(\theta_1^*))] \\
&= \max_{\theta}\ u_q(m-c(\theta)) + v + \frac{\beta\delta\theta}{1-\theta\delta}[u_q(w) + v + (1-\theta)\delta v_1(L=0)] + \beta\delta(1-\theta)v_1(L=0)
\end{aligned}$$

Then, we can take the first order condition of function 47 and solve for the durability choice of the consumer. Let θ^* be the current period choice of the optimal consumption goods.

$$\begin{aligned}
&\textit{[FOC] } \theta\textit{:}\ \ 0 = -u_q'(w-c(\theta^*))c'(\theta^*) + \beta\delta\left[\frac{u_q(w) + v + (1-\theta)\delta v_1(L=0)}{(1-\delta\theta)^2} - \frac{v_1(L=0)}{1-\theta\delta}\right] \\
&\textit{[FOC] } \theta_1\textit{:}\ \ 0 = -u_q'(w-c(\theta^*))c'(\theta^*) + \delta\left[\frac{u_q(w) + v + (1-\theta)\delta v_1(L=0)}{(1-\delta\theta)^2} - \frac{v_1(L=0)}{1-\theta\delta}\right]
\end{aligned} \tag{48}$$

The above FOCs are also the Euler equation that determines the consumption level. However, to compare the durability choice of present bias consumer and time consistent consumers, we do not have to use the exact functional form above.

Lemma 1. *If $u(w-c(\theta))$ is strictly concave in θ and if $\beta < 1$, then $\theta^* < \theta_1^*$*

If $u(w-c(\theta))$ is strictly concave in θ, then $\frac{1}{\beta}u_q'(w-c(\theta^*))c'(\theta^*)$ is strictly decreasing with θ. The derivative of utility with respect to θ for present bias consumer will always be smaller than that of the time consistent consumer due to θ. So the present bias consumer wants to choose a lower durability.

The concavity of $u(w-c(\theta))$ is a reasonable assumption because otherwise we might not have unique points that satisfies the FOC. The results will then be unnecessarily complicated.

8.6.3 Monopolist market, infinite periods, one product, no saving and borrowing

So analyze the monopolist decision, we still use the following assumption:

Assumption 2. *$u_q(w-c(\theta))$ is strictly concave in θ.*

This is the same assumption used in the competitive market.

8.6.4 Time consistent part

If the consumers are time consistent, then the expected utility of buying a light bulb with durability θ and price p will be $U(\theta, p)$ and the utility of owning an light bulb (no need to pay) with durability is $U(\theta, 0)$. $p = 0$ if the consumer owns a working light bulb and there is no need to pay.:

$$\begin{aligned} U_1(\theta, p) =& u_q(w-p) + v + \delta\theta U(\theta, 0) + \delta(1-\theta)U(\theta, p) \\ U_1(\theta, 0) =& u_q(w) + v + \delta\theta U(\theta, 0) + \delta(1-\theta)U(\theta, p) \end{aligned} \tag{49}$$

And we can solve them as:

$$\begin{aligned} U_1(\theta, p) - u_q(w-p) + u_q(w) &= U(\theta, 0) \\ U_1(\theta, p) = u_q(w-p) + v + \delta\theta U(\theta, p) - \delta\theta u_q(w-p) &+ \delta\theta u_q(w) + \delta(1-\theta)U(\theta, p) \\ U_1(\theta, p)(1 - \delta\theta - \delta(1-\theta)) &= u_q(w-p) + v - \delta\theta u_q(w-p) + \delta\theta u_q(w) \\ U_1(\theta, p)(1-\delta) &= u_q(w-p) + v + \delta\theta u_q(w) - \delta\theta u_q(w-p) \\ U_1(\theta, p) &= \frac{1}{1-\delta}[u_q(w-p) + v + \delta\theta u_q(w) - \delta\theta u_q(w-p)] \end{aligned} \tag{50}$$

While if the consumer never buys the product, the expected utility is:

$$U_1(no) = \frac{u_q(w)}{1-\delta} \tag{51}$$

Thus, the willingness to pay for the light bulb is the largest p, such that

$$\begin{aligned} & U_1(\theta, p) \geq U_1(no) \\ & \frac{1}{1-\delta}[u_q(w-p) + v + \delta\theta u_q(w) - \delta\theta u_q(w-p)] \geq \frac{u_q(w)}{(1-\delta)} \\ & u_q(w-p) \geq u_q(w) - \frac{v}{1-\delta\theta} \end{aligned} \tag{52}$$

So, we can solve for p as an explicit function of θ

$$\begin{aligned} p_{TC}(\theta) &= w - u_q^{(-1)}(u_q(w) - \frac{v}{1-\delta\theta}) \\ p'_{TC}(\theta) &= \frac{v\delta}{u'_q(u_q^{(-1)}(u_q(w) - \frac{v}{1-\delta\theta}))(1-\delta\theta)^2} \end{aligned} \tag{53}$$

By taking derivative, we can conclude that $p(\theta)$ is increasing and concave. Then we can calculate the profit of the firm:

$$\begin{aligned}\pi(\theta) &= \max_{\theta}\ p_{TC}(\theta) - c(\theta) + \frac{\delta(1-\theta)(p_{TC}(\theta) - c(\theta))}{(1-\delta)} \\ &= \max_{\theta}\ (1 + \frac{\delta(1-\theta)}{(1-\delta)})(p_{TC}(\theta) - c(\theta))\end{aligned} \tag{54}$$

Then, we have the FOC to solve for the optimal θ:

$$\begin{aligned}\textbf{[FOC]}\ \theta_m\textbf{:}\ \ 0 &= -\frac{\delta}{1-\delta}(p_{TC}(\theta) - c(\theta)) + (1 + \frac{\delta(1-\theta)}{(1-\delta)})(p'_{TC}(\theta) - c'(\theta)) \\ 0 &= -\frac{\delta}{(1-\delta\theta)}(p_{TC}(\theta) - c(\theta)) + p'_{TC}(\theta) - c'(\theta)\end{aligned} \tag{55}$$

8.6.5 Present Biased Consumer

Because of the β discount factor, the willingness to pay for the light bulb should be lower in each period. So, naturally the monopolist will choose a less durable goods at lower price.

The interesting point is the following lemma:

Lemma 2. *If the price is such that the naive consumer always anticipate she will purchase the product in the future, then it cannot be profit maximizing for the firm.*

Lemma 3. *When consumers are present bias, then adding a second product that the consumer will not buy in the first period will increase the profit of the firm. While this policy is not effective for time consistent consumer because they always buy and anticipate to buy the same product.*

In the competitive market, the naive consumer chooses the bulb with durability θ^* while the time consistent consumer chooses durability θ_1^*, is the monopolist going to choose these two levels of durability?

We then write down the value function of consumers. Notice the expected value is the same as the time consistent consumer.

Case 1. the consumer anticipate not to buy the light bulb in the future. (If present biased consumer's wiliness to pay is smaller than that of the time consistent consumers.)

$$\begin{aligned}v(\theta, p) &= u_q(w-p) + v + \beta\delta[\theta U_1(\theta, 0) + (1-\theta)U_1(no)] \\ &= u_q(w-p) + v + \beta[\frac{\theta}{(1-\delta\theta)}[u_q(w) + v + \delta(1-\theta)U_1(no)] + (1-\theta)U_1(no)] \\ &= u_q(w-p) + v + (u_q(w) + v)\frac{\theta\beta}{(1-\delta\theta)} + \frac{\theta\beta}{(1-\delta\theta)}\delta(1-\theta)U_1(no) + \beta(1-\theta)U_1(no) \\ &\geq u_q(w) + v + \beta U_1(no)\end{aligned} \tag{56}$$

Where

$$U_1(no) = \frac{u_q(w)}{1-\delta}$$

$$(57) \quad \begin{aligned} U_1(\theta,0) &= u_q(w) + v + \delta[\theta U(\theta,0) + (1-\theta)U_1(no)] \\ U_1(\theta,0)(1-\delta\theta) &= u_q(w) + v + \delta(1-\theta)U(no) \\ U_1(\theta,0) &= \frac{1}{(1-\delta\theta)}[u_q(w) + v + \delta(1-\theta)U(no)] \end{aligned}$$

Thus, we solve p_{case1} by choosing the largest price that satisfies the following inequality:

$$(58) \quad \begin{aligned} u_q(w - p_{case1}) + v + \beta\delta[\theta U_1(\theta,0) + (1-\theta)U_1(no)] &\geq U_1(no) \\ w - p_{case1} &= u_q^{(-1)}(U_1(no) - v - \beta\delta[\theta U_1(\theta,0) + (1-\theta)U_1(no)]) \end{aligned}$$

$$(59) \quad \begin{aligned} p_{case1} &= w - u_q^{(-1)}(U_1(no) + v + \beta\delta[\theta U_1(\theta,0) + (1-\theta)U_1(no)]) \\ &= w - u_q^{(-1)}(U_1(no) + v + \beta\delta[\frac{\theta}{(1-\delta\theta)}[u_q(w) + v + \delta(1-\theta)U(no)] + (1-\theta)U_1(no)] \end{aligned}$$

While recall that in the time consistent case, the price p_{TC} is obtained by the following function ($U(\theta, p_{TC}(\theta)) = U(no)$):

$$(60) \quad u_q(w - p_{TC}) + v + \delta(\theta U(\theta,0) + (1-\theta)U(no)) \geq U(no)$$

Where θ^*_{TC} is the optimal durability that maximize the monopolist's profit when the consumer is time consistent.

Then, we can calculate the profit function and the first order condition:

$$(61) \quad \pi(\theta) = \max_{\theta}\ p_{case1}(\theta) - c(\theta) + (1-\theta)\delta(p_{case1}(\theta) - c(\theta))/(1-\delta)$$

Then, we have the FOC as

$$(62) \quad \text{[FOC] } \theta_{case1} \quad 0 = -\frac{\delta}{(1-\delta\theta)}(p_{case1}(\theta) - c(\theta)) + p'_{case1}(\theta) - c'(\theta)$$

We know $p_{case1}(\theta,\beta) < p_{TC}(\theta)$ for all θ and β in the range. Because of the future discount factor, the present bias consumer's willingness to pay is strictly smaller for the same durable goods than the time consistent consumers. So, offering the present bias consumer the same durability as θ^*_{TC} and charging p_{TC}, no present bias consumer will buy it because the price is too high. and thus the monopolist would decrease the durability in this case.

Also, because of β, the present bias consumer derives less marginal benefit from increased durability, so increase the durability to more than θ^*_{TC} cannot be profit improving. The monopolist thus wants to choose a lower durability and lower price when the consumer is present bias.

For this case to be true, θ^*_{case1} must be so low that $U(\theta^*_{case1}, p^*_{TC}) < U(no)$, which is always true as I will show below.

Case 2. The consumer anticipates to buy the bulb in the future even when she is time consistent. I will show it is not optimal for monopolist to offer a second good in which the present bias consumer anticipates to buy in the future.

Here, $p_{case2}(\delta)$ is the largest price such that the following is true:

$$u_q(w - p_{case2}) + v + \beta\delta[\theta U(\theta, 0) - \theta U(\theta, p)] \geq u_q(w) \tag{63}$$

However, we know that

$$u_q(w - p_{case2}) + v + \beta\delta[\theta U(\theta, 0) - \theta U(\theta, p)] < u_q(w - p_{case2}) + v + \beta\delta[\theta U(\theta, 0) - \theta U(no)] \tag{64}$$

So, having too high value of continuation payoff is going to harm the price that you can charge today. The monopolist would be better off charging another price that the consumer will not anticipate to purchase in the future.

A side conclusion is that: the monopolist cannot benefit by adding another product that the consumer will not buy, because the best it can do it to increase the continuation payoff, in which if the consumer buys the bulb now, the continuation payoff increases by $\beta\delta(1-\theta)$ while if he does not buy the bulb now, and buy it tomorrow, then the continuation payoff increases by $\beta\delta$, so it decreases the willingness to pay at the current period.

9 Compare monopolist durability and competitive durability

In the linear utility case, we can show the monopolist does not necessarily choose a more or less durable good than the competitive market. Thus, for concave utility function, I can only derive a condition under with the planned obsolescence occurs.

To compare the durability of goods in competitive and monopoly market, we need just need to compare $\frac{\partial \theta^*}{\partial \beta}$ and $\frac{\partial \theta^*_{case1}}{\partial \beta}$, where the former one is the competitive market choice of durability and the latter one is the monopolistic market.

Lemma 4. *For planned obsolescence to occur, the condition is* $\frac{\partial \theta^*}{\partial \beta} < \frac{\partial \theta^*_{case1}}{\partial \beta}$.

We write down the FOCs Competitive market:

$$\begin{aligned} &\text{[FOC] } \theta: \; f_{compete} = u_q'(w - c(\theta^*))c'(\theta^*) - \beta\delta[v_1(\theta^*) + \frac{\partial v_1(\theta^*, L=1)}{\partial \theta^*} - v_1(L=0)] = 0 \\ &\text{[FOC] } \theta_1: \; u_q'(w - c(\theta_1^*))c'(\theta_1^*) = \delta[\frac{u_q(w)}{1-\delta} - \frac{u_q(w - c(\theta_1^*))}{1-\delta} + \frac{1-\theta_1^*}{1-\delta} u_q'(w - c(\theta_1^*))c'(\theta_1^*)] \end{aligned} \tag{65}$$

(66)

$$
\begin{aligned}
v_1(L=0) =& \frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*)) \\
\frac{\partial v_1(\theta, L=1)}{\partial \theta} =& -\frac{\delta}{(1-\delta\theta)^2}[u_q(w) + v + (1-\theta)\delta[\frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*))]] \\
& - \frac{\theta\delta}{1-\theta\delta}[\frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*))] \\
v_1(L=0) =& \frac{v}{1-\delta} + \frac{\delta}{1-\delta}\theta_1^* u_q(w) + \frac{1-\delta\theta_1^*}{1-\delta} u_q(w - c(\theta_1^*))
\end{aligned}
$$

A nice since β_1 and θ^* only appears in the first FOC, many things cancel out and we can directly apply the implicit function theorem and derive:

(67)
$$\frac{\partial \theta^*}{\partial \beta} = \frac{\delta[v_1(\theta^*) + \frac{\partial v_1(\theta^*, L=1)}{\partial \theta^*} - v_1(L=0)]}{\partial f_{compete}(\theta^*)/\partial \theta^*}$$

Similarly, in the monopolistic market, the only variable is θ_m, so we can directly use the FOC to get what we want.

(68)
$$f_{monop}(\theta_m) = (p_{case1}(\theta_m) - c(\theta_m)) + (1-\theta_m)\theta_m(p'_{case1}(\theta_m) - c'(\theta)) = 0$$

(69)
$$\frac{\partial \theta_m}{\partial \beta} = \frac{\frac{\partial p_{case1}(\theta_m)}{\partial \beta} + \frac{\partial (1-\theta_m)\theta_m(p'_{case1}(\theta_m))}{\partial \beta}}{\partial f_{monop}(\theta_m)/\partial \theta_m}$$

So we can study the condition such that $\frac{\partial \theta^*}{\partial \beta} < \frac{\partial \theta^*_{case1}}{\partial \beta}$ is true. However it takes more time to get the economic interpretation.

9.1 Proof of Proposition 4:

The assumption $vQ_1 > c$ ensures that all consumers purchase a new unit of the durable good in period 1. Consider first the case of perfect competition. The first thing to note about perfect competition is that, because of the zero-profit constraint concerning perfect competition, the new-unit price in each period is c. If a consumer does not purchase a new unit in period 2, then in period 3 it has two options. It can choose not to purchase a new unit in period 3 and derive period-3 utility equal to $vQ_1 + Y$. Or it can purchase a new unit in period 3 and derive period-3 utility equal to $vQ_3 + Y - (c - z)$. So the net utility associated with purchasing a new unit in period 3 equals $v(Q_3 - Q_1) - (c - z)$. The parameter restriction $v(Q_3 - Q_1) > c - z$ now ensures that the consumer purchases a new unit in period 3.

Suppose instead a consumer purchases a new unit in period 2. There are again two options.

As before, the consumer can choose not to purchase a new unit in period 3. Now this choice yields period-3 utility equal to $vQ_2 + Y$. Or the consumer can purchase a new unit in period 3 and derive period-3 utility equal to $vQ_3 + Y - (c - z)$. So the net utility associated with purchasing a new unit in period 3 now equals $v(Q_3 - Q_2) - (c - z)$. The parameter restriction $c - z > v(Q_3 - Q_2)$ now tells us that the consumer does not purchase a new unit in period 3.

Now consider period 2. A consumer has two options. The consumer could choose not to purchase a new unit which from above means the consumer purchases a new unit in period 3. From the standpoint of period 2, the consumer's discounted utility over periods 2 and 3 equals $vQ_1 + Y + \delta[vQ_3 + Y - (c - z)]$. Or the consumer could choose to purchase a new unit in period 2 which from above means the consumer does not purchase a new unit in period 3. From the standpoint of period 2, the consumer's discounted utility over periods 2 and 3 now equals $vQ_2 + Y - (c - z) + \delta(vQ_2 + Y)$. Setting the two expressions equal to each other and solving for Q_2 yields that all consumers purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_2^*$ ($Q_2 < Q_2^*$). Also, i) immediately follows given as stated earlier that perfect competition tells us that the price for a new unit is c in each period.

We now consider monopoly. If the monopolist does not sell new units in period 2, then in period 3 it has two options. It can choose not to sell new units in period 3 and earn period-3 profit equal to zero. Or it can sell new units in period 3 and charge consumers their willingness to pay which equals $v(Q_3 - Q_1) + z$. Given the parameter restriction $v(Q_3 - Q_1) > c - z$, the monopolist sells new units in this case and earns $N[v(Q_3 - Q_1) - (c - z)]$.

Suppose instead consumers purchase new units in period 2. There are again two options for the monopolist. It can again choose not to sell new units in period 3 and earn period-3 profit equal to zero. Or it can sell new units in period 3 and charge consumers their willingness to pay which equals $v(Q_3 - Q_2) + z$. Given the parameter restriction $c - z > v(Q_3 - Q_2)$, the monopolist does not sell new units in this case and earns zero in period 3.

Now consider period 2. The monopolist has two options. The monopolist could choose not to sell new units which from above means the monopolist sells new units in period 3. From the standpoint of period 2, the discounted value for monopoly profit in this case equals $\delta[v(Q_3 - Q_1) - (c - z)]$. Or the monopolist could choose to sell new units in which case it charges consumers their willingness to pay. Given that the monopolist extracts all incremental surplus in period 3 if a consumer does not purchase a new unit in period 2, consumer willingness to pay for a new unit in period 2 equals $(1 + \delta)v(Q_2 - Q_1) + z$. So from the standpoint of period 2, the discounted value for monopoly profit in this case equals $(1 + \delta)v(Q_2 - Q_1) - (c - z)$. Setting the two expressions equal to each other and solving for Q_2 yields that all consumers purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_2^*$ ($Q_2 < Q_2^*$).

Consider monopoly and the case $Q_2 > Q_2^*$. From above we know the period-2 price equals $(1 + \delta)v(Q_2 - Q_1) + z$. Now consider period 1. Using arguments like those above, if consumers do not purchase new units in period 1, then the monopolist extracts all surplus in subsequent periods. Similarly, if consumers purchase new units in period 1, then from above we know the monopolist extracts all the incremental surplus in subsequent periods. So in period 1 the monopolist charges consumers their period-1 willingness to pay which equals $(1 + \delta + \delta^2)vQ_1$ and all the surplus is captured by the monopolist. This proves ii), while iii) follows from a similar argument.

9.2 Proof of Proposition 5:

Consider first the case of perfect competition. From the proof of Proposition 4 we know: i) all consumers purchase a new unit in period 1; ii) the new-unit price in each period is c; iii) if a consumer does not purchase a new unit in period 2, then the consumer purchases a new unit in period 3; and iv) if a consumer purchases a new unit in period 2, then the consumer does not purchase a new unit in period 3.

Now consider period 2. A consumer has two options. The consumer can choose not to purchase a new unit which from above means the consumer purchases a new unit in period 3. From the standpoint of period 2, the consumer's perceived discounted utility over periods 2 and 3 equals $vQ_1 + Y + \beta\delta[vQ_3 + Y - (c - z)]$. Or the consumer could choose to purchase a new unit in period 2 which from above means the consumer does not purchase a new unit in period 3. From the standpoint of period 2, the consumer's perceived discounted utility over periods 2 and 3 equals $vQ_2 + Y - (c - z) + \beta\delta(vQ_2 + Y)$. Setting the two expressions equal to each other and calling the resulting value for $Q_2, Q_{2,P'}, yields Q_{2,P'} < Q_2^*$, and that consumers purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_{2,P'}(Q_2 < Q_{2,P'})$. Also, i) immediately follows given as stated earlier that perfect competition tells us that the price for a new unit is c in each period, that Q_2^* is the first-best value, and that in perfect competition all surplus goes to consumers.

We now consider monopoly. From the proof of Proposition 4 we know: i) all consumers purchase a new unit in period 1; ii) if the monopolist does not sell new units in period 2, then it sells new units in period 3 and period-3 profit equals $N[v(Q_3 - Q_1) - (c - z)]$; and iii) if the monopolist sells new units in period 2, then it does not sell new units in period 3 and period-3 profit equals zero.

Now consider period 2. The monopolist has two options. The monopolist could choose not to sell new units which from above means the monopolist sells new units in period 3. From the standpoint of period 2, the discounted value for monopoly profit in this case equals $\delta[v(Q_3 - Q_1) - (c - z)]$. Or the monopolist could choose to sell new units in which case it charges consumers their willingness to pay. Given that the monopolist extracts all incremental surplus in period 3 if a consumer does not purchase a new unit in period 2, consumer willingness to pay for a new unit in period 2 equals $(1 + \beta\delta)v(Q_2 - Q_1) + z$. So from the standpoint of period 2, the discounted value for monopoly profit in this case equals $(1 + \beta\delta)v(Q_2 - Q_1) - (c - z)$. Solving for $Q_{2,M'}$, which is the value for Q_2 that equates the expressions, yields $Q_{2,M'} > Q_2^*$, and that consumers purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_{2,M'}$ ($Q_2 < Q_{2,M'}$).

Consider monopoly and the case $Q_2 > Q_{2,M'}$. From above we know the period-2 price equals $(1 + \beta\delta)v(Q_2 - Q_1) + z$. Now consider period 1. Using arguments similar to arguments above, consumers will anticipate that if they do not purchase new units in period 1, then the monopolist will extract all the surplus in subsequent periods. Similarly, consumers will also anticipate that if they purchase new units in period 1, then the monopolist will extract all the surplus in subsequent periods. So in period 1 the monopolist charges consumers their period-1 willingness to pay which equals $(1 + \beta\delta + \beta\delta^2)vQ_1$. Note that, because the prices are below prices when $\beta = 1$, the surplus is shared between the monopolist and the consumers, where the consumer surplus is higher than when $\beta = 1$ and monopoly profit is lower than when $\beta = 1$. This proves ii), while iii) follows from a similar argument.

9.3 Proof of Proposition 6:

The assumption $vQ_1 > c$ ensures that all consumers purchase a new unit of the durable good in period 1. Consider first the case of perfect competition. The first thing to note about perfect competition is that, because of the zero-profit constraint, the new-unit price in each period is c. If a consumer does not purchase a new unit in period 2, then in period 3 she can either purchase a new unit or not. Using arguments found in the proof of Proposition 4 we can show that the net utility associated with purchasing a new unit in period 3 equals $v(Q_3 - Q_1) - (c - z)$. Given the parameter restriction $c - z > v(Q_3 - Q_1)$, consumers would choose not to purchase new units in period 3. If a consumer purchases a new unit in period 2, then in period 3 it again can either purchase a new unit or not. Using arguments found in Proposition 4 we can show that the net utility associated with purchasing a new unit in period 3 equals $v(Q_3 - Q_2) - (c - z)$. Given the parameter restriction $c - z > v(Q_3 - Q_1)$ and $Q_2 > Q_1$, we again have that consumers would choose not to purchase new units in period 3. That is, independent of period-2 behavior, consumers do not purchase new units in period 3.

Now consider period 2. A consumer has two options. The consumer could choose not to purchase a new unit and from above we know that the consumer will then choose to not purchase a new unit in period 3. From the standpoint of period 2, the consumer's discounted utility over periods 2 and 3 equals $(1 + \delta)(vQ_1 + Y)$. Or the consumer could choose to purchase a new unit in period 2 and from above we know that the consumer will again choose to not purchase a new unit in period 3. From the standpoint of period 2, the consumer's discounted utility over periods 2 and 3 now equals $(1+\delta)(vQ_2+Y) - (c-z)$. Setting the two expressions equal to each other and solving for Q_2 yields that all consumers purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q^{**}$($Q_2 < Q_2^{**}$). Also, i) immediately follows given as stated earlier that perfect competition tells us that the price for a new unit is c in each period.

We now consider monopoly. If the monopolist does not sell new units in period 2, then in period

3 it can either sell new units or not. As shown in the proof of Proposition 4, if it does not sell new units then it earns zero in period 3, while if it sells new units then it earns $v(Q_3 - Q_1) - (c - z)$. Given the parameter restriction $c - z > v(Q_3 - Q_1)$, the monopolist would choose not to sell new units in period 3. If instead the monopolist sells new units in period 2, then again in period 3 it can either sell new units or not. As shown in the proof of Proposition 4, if it does not sell new units then it earns zero in period 3, while if it sells new units then it earns $v(Q_3 - Q_2) - (c - z)$. Given the parameter restriction $c - z > v(Q_3 - Q_1)$ and $Q_2 > Q_1$, the monopolist would choose not to sell new units in period 3. That is, independent of period-2 behavior, the monopolist does not sell new units in period 3.

Now consider period 2. The monopolist has two options. The monopolist could choose not to sell new units which from above means the monopolist does not sell new units in either period 2 or period 3. From the standpoint of period 2, the discounted value for monopoly profit in this case equals zero. Or the monopolist could choose to sell new units in which case it charges consumer willingness to pay. Given that the monopolist does not sell new units in period 3 whether or not it sells new units in period 2, consumer willingness to pay for a new unit in period 2 equals $(1 + \delta)v(Q_2 - Q_1) + z$. So from the standpoint of period 2, the discounted value for monopoly profit in this case equals $(1 + \delta)v(Q_2 - Q_1) - (c - z)$. Setting the two expressions equal to each other and solving for Q_2 yields that all consumers purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q_2^{**}$ ($Q_2 < Q_2^{**}$). Finally, given this, ii) and iii) follow from arguments similar to those provided in the proof of Proposition 4.

9.4 Proof of Proposition 7:

Consider first the case of perfect competition. From the proof of Proposition 6 we know: i) all consumers purchase a new unit in period 1; ii) the new-unit price in each period is c; and iii) a consumer does not purchase a new unit in period 3 independent of whether the consumer purchases or does not purchase a new unit in period 2.

Now consider period 2. A consumer has two options. The consumer can choose not to purchase a new unit which from above means the consumer does not purchase a new unit in period 3. From the standpoint of period 2, the consumer's perceived discounted utility over periods 2 and 3 equals $(1 + \beta\delta)(vQ_1 + Y)$. Or the consumer could choose to purchase a new unit in period 2 which from above means the consumer does not purchase a new unit in period 3. From the standpoint of period 2, the consumer's perceived discounted utility over periods 2 and 3 equals $(1+\beta\delta)(vQ_2+Y)-(c-z)$. Setting the two expressions equal to each other and calling the resulting value for Q_2 , $Q_{2,P''}$, yields $Q_{2,P''} > Q_2^{**}$, and that consumers purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q_{2,P''}$ ($Q_2 < Q_{2,P''}$). Also, i) immediately follows given as stated earlier that perfect competition tells us that the price for a new unit is c in each period, that Q_2^{**} is the first-best value, and that in perfect competition all surplus goes to consumers.

We now consider monopoly. From the proof of Proposition 6 we know: i) all consumers purchase a new unit in period 1; ii) if the monopolist does not sell new units in period 2, then it does not sell new units in period 3 and period-3 profit equals zero; and iii) if the monopolist sells new units in period 2, then it does not sell new units in period 3 and period-3 profit equals zero.

Now consider period 2. The monopolist has two options. The monopolist could choose not to sell new units which from above means the monopolist does not sell new units in period 3. From the standpoint of period 2, the discounted value for monopoly profits over periods 2 and 3 equals zero. Or the monopolist could choose to sell new units in which case it charges consumers their willingness to pay. Given that the monopolist does not sell new units in period 3 whether or not consumers purchase new units in period 2, consumer willingness to pay for a new unit in period 2 equals $(1 + \beta\delta)v(Q_2 - Q_1) + z$. So from the standpoint of period 2, the discounted value for monopoly profit in this case equals $(1 + \beta\delta)v(Q_2 - Q_1) - (c - z)$. Solving for $Q_{2,M''}$ which is the value that equates this expression with zero yields $Q_{2,M''} > Q_2^{**}$, and that consumers purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q_{2,M''}$ ($Q_2 < Q_{2,M''}$). Finally, ii) and iii) follow from the same arguments used to prove ii) and iii) in the proof of Proposition 5.