A BEHAVIORAL APPROACH TO DURABILITY CHOICE, NEW-PRODUCT INTRODUCTIONS, AND PLANNED OBSOLESCENCE *

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Abstract

Observation of real-world markets suggests that many products are produced at below-efficient built-in durability levels, and/or new products are introduced quickly which inefficiently reduces the useful life of durable products. Most of the prior literature explains these observations employing monopoly/market power models, but these behaviors are also found in competitive markets. We show that present-biased consumer preferences can cause these outcomes by affecting equilibrium durability and product introduction timing. Our analysis further reveals that market power can intensify these distortions. We discuss various real-world applications including the famous Phoebus light bulb cartel and product recalls in the medical device sector.

Keywords: durability, new-product introductions, present bias

JEL codes: D4, L4

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1 INTRODUCTION

There are numerous examples of markets in which product durability seems below what is easily achievable. This could be because of new-product introductions that make used units obsolete as in the video game or fashion industries, or because the product has a low level of built-in durability as seems to have been the case for light bulbs prior to the recent regulatory changes. Previous theoretical literature on the subject has mostly focused on monopoly/market power models to explain the behavior. But many of the real-world examples where the behavior is observed seem closer to competition than to monopoly. In this paper, we explore the extent to which time-inconsistent/present-biased preferences, as first put forth by Strotz (1955) and extended, for example, by Lowenstein and Prelec (1992), Laibson (1997), and O'Donoghue and Rabin (1999), can explain inefficiently low levels of built-in durability and frequent new-product introductions in both competitive and monopoly settings.

Why would present-biased consumer preferences be important for durability choice and the frequency of new-product introductions? We know that, in the absence of the ability to commit, present-biased consumer preferences can lead to savings levels below efficient levels, as discussed initially by Strotz (1955). The main point of our paper is that purchasing a good with a longer useful lifetime is similar to increasing savings in that it increases future consumption at the expense of current consumption. So, similar to present-biased preferences resulting in below efficient savings levels, such preferences can also result in inefficiency concerning built-in durability choice and the frequency of new-product introductions.

In the first part of our analysis, we consider an infinite-period model in which new and used units are perfect substitutes in consumption, where durability choice concerns the probability that a unit lasts an extra period. Consumers have present-biased preferences of the type modeled by Laibson (1997) and O'Donoghue and Rabin (1999), where we initially assume consumers are naïve meaning that a consumer in one period does not anticipate that she will exhibit present bias when making consumption decisions in future periods. Also, consumers in each period derive utility from the consumption of durable goods and from the consumption of "other goods" which we denote as the numeraire good. Also, in the main part of the analysis we assume that consumers derive constant marginal utility from consumption of the numeraire good.

In this model, given perfect competition, firms produce units characterized by the first-best durability level in the absence of present-biased preferences, but produce units with less durability when consumers exhibit present bias and are naïve The logic is that more durable units cost more to produce which under perfect competition means they have a higher price. So by purchasing less durable units that cost less, a consumer can shift consumption from future periods to the current period. With time-consistent preferences, the incentive to shift consumption from the future to the present is determined by the discount factor common to all players in the game, with the result that firms produce and sell units with the efficient durability level. But when consumers have

present-biased preferences and are naïve, the incentive for consumers to move consumption from the future to the present exceeds that suggested by the common discount factor, and the result is a durability level below the first best.

We also consider equilibrium behavior given monopoly rather than perfect competition. Here we find results similar to what we found in the competitive case. That is, when consumers are characterized by time-consistent preferences, the monopolist chooses the first-best durability level. But when consumers exhibit present bias and are naïve, then the monopolist produces and sells units that are characterized by durability below the first-best level. The logic here is that, given present bias, consumer willingness to pay in any current period for current-period services is higher than willingness to pay for these services in any previous period. As a result, a durable-goods monopolist can increase profits by producing less durable units since that increases the frequency with which current-period services are purchased in the current period.

Note that our finding that, given perfect competition or monopoly, when consumers have time-consistent preferences durability is efficient is consistent with Swan's (1970,1971) classic analysis of the durability issue, as discussed in detail later. That is, given there is not a commitment issue in our model and new and used units are perfect substitutes in consumption, Swan's logic tells us that durability should be efficient given any market structure. What we show which is new to the literature is that present-biased consumer preferences causes Swan's logic to break down with the result that durability is below the efficient level given both perfect competition and monopoly.

We also compare equilibrium durability, given present-biased and naïve consumers, under perfect competition and monopoly. Here we find that equilibrium durability is lower under monopoly than under perfect competition. In other words, the incentive for the monopolist to reduce durability due to higher consumer willingness to pay for current-period services is higher than the incentive for reduced durability in the competitive case due to the lower incentive for savings. The logic is that, because willingness to pay is higher than the cost of producing the durable good, the monopolist has a higher incentive to distort the durability decision.

We then consider how results change when consumers are sophisticated rather than naïve, where a sophisticated consumer anticipates in each period that in making purchase decisions in later periods, she will exhibit present bias. Here we find that most of the qualitative results for the naïve consumer case continue to hold. For example, it is still the case that, under both perfect competition and monopoly, the equilibrium durability level is below the efficient level. We find, however, that the distortion in the case of perfect competition is smaller when consumers are sophisticated. In contrast, in the case of monopoly, behavior is the same in the sophisticated and naïve consumer cases. The logic in the case of perfect competition is that sophisticated consumers anticipate that future behavior will be time inconsistent, and this reduces their incentive to purchase units with below-efficient durability. In the case of monopoly, both sophisticated and naïve consumers anticipate that the monopolist will extract all the surplus in future periods, and the result is that equilibrium behavior is the same in the two cases.

In the second part of our analysis, we focus on how present-biased consumer preferences affect new-product introductions that make used units obsolete. In particular, we consider a three-period model in which there is exogenous technological progress over time, so a new product introduced in period two is of higher quality than period one production, and a new product introduced in period three is of higher quality than a new product introduced in period two. Our main focus is whether present bias can result in planned obsolescence due to inefficiently early new product introductions, i.e., a new product being introduced in period 2 when a period three introduction would be more efficient. Note that in this model, given the three-period structure and the parameterizations we focus on, there are only minor differences between the naïve and sophisticated consumer cases. The results described below are valid for both cases, although in our formal analysis we focus on the naïve consumer case.

Here we show that, given both perfect competition and monopoly, when consumers have time-consistent preferences, a new product is introduced in period two rather than period three whenever introducing a new product in period two is efficient, i.e., when the introduction is such that the discounted change in gross consumer utility over periods two and three exceeds the discounted change in costs. In contrast, when consumers have present-biased preferences, under perfect competition there are parameterizations for which a new-product introduction occurs in period two rather than period three even when this condition is not satisfied.

We show that this result can occur given parameterizations in which it is efficient to introduce a new product only in period three, but a new-product introduction in period two stops a new product from being introduced in period three, due to the reduced incremental quality associated with a period-three introduction. The logic is that in period two present-biased consumers place a low value on consuming a higher quality product in period three, so a present-biased consumer does not care that a new-product introduction in the second period stops the introduction of an even higher quality new product in period three. The result is that in perfect competition with present-biased consumers, a new product may be introduced in period 2 when it is efficient to delay the new product introduction to period 3.

In this analysis, we also show that there are parameterizations for which present bias results in no new product introduction in period 2 when there is a period-2 introduction in the absence of present bias. For example, this can occur under both perfect competition and monopoly when there would be no period-3 introduction independent of whether or not there is a period-2 introduction. The logic for this result follows from the defining feature of what it means to be present biased. That is, for these parameterizations the benefit of a new-product introduction in period two is higher consumer utility in periods two and three due to the consumption of a higher quality unit of the durable good. Present-biased consumers in period two, however, place little value on any extra utility associated with consuming a higher quality unit in period three. As a result, for these parameterizations time-consistent consumers can find it optimal to purchase new higher quality units in period two, while present-biased consumers do not.

At the end of the paper, we discuss potential applications of our analysis. One of our applications concerns the well-known Phoebus light bulb cartel established in 1925 which lasted more than a decade. The cartel raised prices and reduced the life expectancy of the light bulbs produced from approximately 1,800 hours to approximately 1,200 hours. Some authors have argued that this reduction resulted in the durability of the bulbs being below the efficient level (see, for example, Krajewsi (2014)). As we discuss in more detail later, prior theories of reduced durability and planned obsolescence are not good explanations for this aspect of the cartel's behavior. However, the analysis in Section 3 provides a theoretical foundation for why the cartel may have found it profitable to practice a type of planned obsolescence. We also discuss how our theory helps explain recent regulatory behavior in the light bulb industry and product recall behavior in the medical device industry.

Overall, this paper contributes to the literature concerning real-world applications of the idea that in many settings consumers exhibit present bias. This includes various behaviors related to savings (see, for example, Strotz (1955)), procrastination (see, for example, O'Donoghue and Rabin (1999)), and borrowing at high-interest rates on credit cards (see, for example, Meier and Sprenger (2010)). Given the importance of durability choice and new-product introductions in many markets, we feel that our results concerning how present bias affects behavior in durable-goods markets may be one of the more important applications of the present-bias idea.

The outline for the paper is as follows. Section 2 discusses the related literature. Section 3 presents and analyzes our model of built-in durability. Section 4 presents and analyzes our model of new-product introductions. Section 5 discusses a potential application of our theoretical findings. Section 6 provides concluding remarks.

2 LITERATURE REVIEW

This paper mostly contributes to two literatures: i) papers on durability choice, new-product introductions, and planned obsolescence; and ii) papers concerning real-world applications of the idea that individuals have present-biased preferences, especially the small set of papers related to durable-goods markets. We start by discussing the literature on durability choice, new-product introductions, and planned obsolescence, and then discuss the literature concerning present bias. See Waldman (2003) for a survey that discusses the former literature, and O'Donoghue and Rabin (2015) for a survey that discusses the latter.

In a series of influential papers in the early 1970s, Peter Swan showed that under a wide range of settings both perfect competition and monopoly result in the efficient choice of durability (see, for example, Swan (1970,1971) and Sieper and Swan (1973)). The basic idea is that, if consumers care only about the services provided and not durability directly, then under both market structures

¹Swan's papers improve on earlier analyses of the durability issue such as Kleiman and Ophir (1966), Levhari and Srinivasan (1969), and Schmalensee (1970).

firms choose the cost minimizing or socially optimal level of durability. He showed this result both when new and used units are perfect substitutes, and when units depreciate or deteriorate with age, but some number of used units is a perfect substitute for a new unit.

A number of papers investigate settings in which Swan's argument does not apply. Probably the two most important are monopoly/market power arguments due to Bulow (1986), Waldman (1996a), and Hendel and Lizzeri (1999a).² In his 1986 paper, Bulow builds on Coase (1972) and Bulow (1982), and shows that a monopolist may reduce durability to partially avoid the costs of time inconsistency which arise when a durable-goods monopolist cannot commit to future prices or production levels. Waldman (1996a) and Hendel and Lizzeri (1999a) show that, if used units are lower quality than new units and durability choice affects the speed of quality deterioration, a durable-goods monopolist may choose durability below the efficient level in order to more effectively price discriminate across consumers with different valuations on product quality.³ In contrast to these market power arguments, we focus on a behavioral approach concerning consumer preferences which can apply in both market power and competitive settings.

A related literature focuses on reasons why durable-goods producers may reduce the effective durability of a product by frequent introduction of new units that make used units obsolete. One argument in this literature, due initially to Waldman (1993,1996b) and Choi (1994), is that time inconsistency can cause a monopoly producer to practice this type of planned obsolescence, i.e., a durable-goods monopolist that sells its output will not internalize how new-product introductions affect the value of used units with the result being inefficiently frequent new-product introductions. Another argument concerns the role that fashion plays in signaling status, income or related attributes in games of social interaction (see, for example, Pesendorfer (1995)). Our approach, in contrast, depends neither on market power, nor the product being easily observable by other market participants, as is required in signaling games of social interaction.⁴

As indicated earlier, the paper also contributes to the literature on time-inconsistent/present-biased preferences, as originally put forth in the seminal analyses of Strotz (1955), Lowenstein and Prelec (1992), and Laibson (1997). We contribute to the part of the literature that employs the present-bias assumption to explain various real-world behaviors. In particular, we explore the implications of present-biased preferences for behavior in durable-goods markets when durability is an endogenous choice. Since purchasing a durable good is similar to savings in that the behavior trans-

²See Schmalensee (1979) for a survey of earlier literature investigating settings in which Swan's argument does not apply.

³See Anderson and Ginsburgh (1994) for an earlier related analysis. Also, note that this argument builds on the seminal analysis of Mussa and Rosen (1978) concerning how a monopolist chooses prices and qualities when selling a product line.

⁴A third argument, due to Grout and Park (2005), is that new-product introductions that make used units obsolete can arise in competitive settings to reduce problems due to adverse selection concerning secondhand-market trade. Our analysis abstracts away from any problems concerning adverse selection and secondhand-market trade since in our analysis consumers are all identical so secondhand-market trade is not a factor (also, in our model used-unit quality is not stochastic which is a second reason adverse selection is not a factor). Note that the first two applications we discuss in Section 5 concern an industry, i.e., the light bulb industry, characterized by little or no secondhand-market trade.

lates into purchasing a good used for consumption in a future period, it seems intuitive that present bias will be important for equilibrium durability choice given that it is important for understanding various aspects of savings behavior. We formally investigate equilibrium implications.

Note that a few previous papers have considered the implications of present-biased preferences for behavior in durable-goods markets. For example, Nocke and Peitz (2003) focus on how present bias affects secondary markets for durable goods, Bar-Gill and Hayashi (2021) consider debt financing used to purchase durable goods when consumers are present biased, while Kang and Kang (2022) show how purchasing durable goods can be used as a commitment device by sophisticated consumers characterized by present bias. All of these papers take the degree of product durability as given rather than endogenously determined. We instead focus on the implications of present bias for equilibrium durability - whether built-in or determined by the speed of new-product introductions - which is a classic issue in the industrial organization literature.

3 PRESENT BIAS AND BUILT-IN DURABILITY CHOICE

In this section, we consider how present bias affects equilibrium built-in durability choice given both perfect competition and monopoly. In the first subsection we present the model, while in the second we analyze the model under the two market structures given constant marginal utility for the numeraire good. In the third subsection, we analyze how results change given decreasing marginal utility for the numeraire good. Note that our analysis includes a comparison of durability choice across perfect competition and monopoly given present-biased consumer preferences.

A) The Model

We consider an infinite-period discrete-time model. Consumers are infinitely lived, which is also the case for the single firm in the monopoly case and multiple firms in the case of perfect competition. Within a period new and used units that are in good working order are perfect substitutes in consumption, where units can be different levels of durability. To be precise, θ denotes the durability level, where a unit produced in any period t has a probability $(1-\theta)$ of lasting a single period, a probability $\theta(1-\theta)$ of lasting two periods, a probability $\theta^2(1-\theta)$ of lasting three periods, etc. We also assume that there are no fixed costs of production, while the marginal cost of production increases with the durability of the unit. For a given durability, the marginal cost is constant for any number of units produced. In particular, producing x units of durability θ costs $xc(\theta)$, where $c(\cdot)$ is differentiable everywhere in the range [0,1] and satisfies c(0) = C > 0, $c(1) = \infty$, c'(0) = 0, $c'(\theta) > 0$ for all $0 < \theta \le 1$, and $c''(\theta) > 0$ for all $0 < \theta \le 1$. The assumption $c(1) = \infty$ combined with convexity ensures a unique first-best solution.

There are N identical consumers, where consumer income each period is given by w. Consumers spend their income each period purchasing either zero or one unit of the durable good, while the remaining income is spent on purchasing the numeraire good (other goods) - we do not allow savings

or borrowing (see the Conclusion for a discussion). We also assume that w is sufficiently large that in each setting we consider a consumer purchases a positive amount of the numeraire good in every period.

Utility for representative consumer i in period t, $\mu_{i,t}$, is given by equation (1)

(1)
$$\mu_{i,t} = L_{i,t}V + f(x_{i,t})$$
$$s.t. \ w_{i,t} + x_{i,t} = w$$

In equation 1, $L_{i,t} = 1(0)$ when the consumer owns (does not own) a working unit of the durable good in period t, where the consumer can own a working unit by either purchasing a new unit in period t or by owning a working unit purchased in a previous period.⁵ V is the consumer's gross utility from owning a working unit of the durable good in any period, V > C. $w_{i,t}$ is the consumer's expenditure on durable goods in period t. $x_{i,t}$ is the number of units of the numeraire good consumed in period t, where $f(x_{i,t})$ captures the consumer's gross utility in period t from consumption of the numeraire good. We assume $f(x_{i,t})$ is strictly increasing and weakly concave, where much of the focus below is on the special case where $f(x_{i,t})$ is linear, i.e., $f''(x_{i,t}) = 0$ for all $x_{i,t}$.

In each period t, consumer i chooses behavior consistent with maximizing her perception of expected utility over the remainder of her lifetime. To be precise, in each period t consumer i maximizes $U_{i,t}$ which is given in equation (2).

(2)
$$U_{i,t} = \mu_{i,t} + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \mu_{i,t}$$

 δ is the discount factor, $0 < \delta < 1$, and including β means we allow for the possibility that consumers are present biased, where we incorporate present bias by employing the now standard assumption of hyperbolic discounting. In particular, we consider both the case in which consumers are time consistent, i.e., $\beta = 1$, and the case of present bias, i.e., $0 < \beta < 1$. Note that in the next subsection we start by assuming constant marginal utility for the numeraire good and that consumers are naïve in the sense defined by O'Donoghue and Rabin (1999), i.e., in making decisions in each period t, consumers ignore that in future periods their behavior will be present biased. O'Donoghue and Rabin (2015) argue that this is the more realistic assumption. Then, we solve the constant marginal utility case with sophisticated consumers. In the last subsection we consider behavior when there is decreasing marginal utility for the numeraire good. Also, firms are not present biased which means for firms discounting is determined solely by the discount factor δ .

The timing of the game is as follows. At the beginning of each period, every consumer who owned a working unit in the previous period observes whether or not the unit will work this period

⁵We do not allow second hand-market trade, but that has no significant effect on equilibrium behavior given consumers are identical. See footnote 4 for a related discussion.

(note, equilibrium behavior is independent of whether this observation is public or private). Every durable-goods producer then chooses a durability level for its output and a price. Each consumer then makes period-t purchases, where any income not spent on purchasing a unit of the durable good is spent on the numeraire good. Our focus is Markov Perfect equilibria.

B) Constant Marginal Utility for the Numeraire Good

In this subsection, we analyze the model presented in the previous subsection given constant marginal utility for the numeraire good. The main point of the analysis in this subsection is that, given time-consistent preferences, durability is efficient in both the perfect competition and monopoly cases, which is consistent with Swan's classic analysis. But a new result relative to the existing durable goods literature is that present-biased preferences result in Swan's logic not applying, and durability choice under both perfect competition and monopoly below the efficient level.

We start by characterizing the first best. The first best in this case is the behavior that would be chosen by a social planner who decides production levels in each period, how production is allocated across consumers, and who maximizes the expected discounted value starting in period 1 of realized consumer gross utilities minus the costs of production.⁶ Below θ^* denotes the first-best durability level.

It is easy to show that in the first best in our model every consumer is allocated a new durable unit in any period in which the consumer does not own a working durable unit at the beginning of the period, while θ^* is the value that minimizes the expected cost of having all consumers consume a unit of the durable good in every period. That is, since in the first best all consumers consume a unit of the durable good in each period, maximizing social welfare reduces to finding the durability level that minimizes the cost of achieving this result.

We first consider how the model works given constant marginal utility for the numeraire good and consumers who are time consistent.

Proposition 1. Suppose consumers have constant marginal utility for the numeraire good, i.e., f'' = 0. If consumers are time consistent, i.e., $\beta = 1$, then under both perfect competition and monopoly all consumers consume a durable unit in each period and firms produce units of durability θ^* . Also, i) and ii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price for a durable good in each period is $c(\theta^*)$ and all surplus is received by consumers.
- ii) Given monopoly, the new-unit price for a durable good in each period equals $w f'^{-1}(f(w) \frac{V}{1-\delta\theta})$ and all surplus is received by the monopolist.

⁶In our analysis of the first best, we aggregate consumer utilities across periods setting $\beta = 1$. This is standard for models of hyperbolic discounting. This is also how we calculate consumer surplus in our present-bias analysis. See O'Donoghue and Rabin (1999) for a discussion.

Proposition 1 1 demonstrates that our model is consistent with Swan's conclusion concerning durability choice given constant marginal utility for the numeraire good and time-consistent preferences. That is, given these two conditions are satisfied, then under both perfect competition and monopoly the firms choose the durability level that minimizes the cost of producing the equilibrium flow of services, i.e., the socially optimal durability level.

The next part of the analysis explores how results change given consumers who are present biased and naïve rather than time consistent.

Proposition 2. Suppose consumers have constant marginal utility for the numeraire good, i.e., f'' = 0. If consumers are present biased, i.e., $\beta < 1$, and naïve, then under both perfect competition and monopoly, all consumers consume a durable unit in each period and firms produce units of durability $\theta'_{P,N}$, $\theta'_{P,N} < \theta^*$, in the case of perfect competition, and $\theta'_{M,N}$, $\theta'_{M,N} < \theta^*$, in the case of monopoly. Also, i) and ii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price in each period is $c(\theta'_{P,N})$. Also, both consumer welfare and social welfare are below the values when $\beta = 1$ because of the distortion in the durability level (while producer surplus is unchanged since it equals zero in each case).
- ii) Given monopoly, the new-unit price in each period is $w f'^{-1}\left(f(w) \left[\frac{V}{1-\beta\delta\theta}\right]\right)$. In this case, monopoly profit is below the value when $\beta = 1$, while consumer surplus can be higher or lower than when $\beta = 1$.

The main result in Proposition 2 is that with constant marginal utility for the numeraire good and present-biased consumers who are naïve, given both perfect competition and monopoly, firms produce units that are less durable than in the first best, or equivalently, less durable than when consumers have time-consistent preferences. The logic for this result in the perfect competition case is as follows. More durable units cost more to produce, which in the case of perfect competition translates into a higher new-unit price. The return to having more durable units is less frequent need to buy replacement units which, in turn increases future consumption of the numeraire good. But a present-biased consumer places less value on future consumption of the numeraire good, so has a smaller willingness to pay for incremental increases in durability which translates into a lower equilibrium durability level.

The logic for the monopoly result is related. The monopolist charges consumers their willingness to pay for a new unit given they do not currently own a working unit. Present bias means consumers value the future less and are thus less willing to pay for incremental increases in durability. The result is lower durability than equilibrium durability given time-consistent consumers. Another way to look at this result is that, because of present bias, consumer willingness to pay for current period services is higher in any current period than in earlier periods. Reducing durability increases the probability that current period durable-goods services are purchased in the current period, which takes advantage of higher willingness to pay and thus increases monopoly profitability.

The proposition also contains a number of other interesting results. For example, present bias hurts consumers but not firms under perfect competition, while under monopoly present bias hurts the monopolist and has an ambiguous effect on consumers. The result concerning perfect competition follows given that the distortion concerning the equilibrium durability choice hurts social welfare, and we know that under perfect competition all surplus goes to the consumers. That present bias hurts monopoly profits follows given that present bias reduces consumers' willingness to pay and thus monopoly profitability, holding durability fixed.

We next analyze the model given consumers are present biased as above, but now we assume the consumers are sophisticated rather than naïve. The definition of a sophisticated consumer is that in each period the consumer correctly anticipates that in future periods she will exhibit present bias. Note that in the analysis that follows we focus on stationary Markov Perfect equilibria.⁷

Proposition 3. Suppose consumers have constant marginal utility for the numeraire good, i.e., f''=0. If consumers are present biased, i.e., $\beta<1$, and sophisticated, then under both perfect competition and monopoly, all consumers consume a durable unit in each period and firms produce units of durability $\theta'_{P,S}$, $\theta'_{P,N}<\theta'_{P,S}<\theta^*$, in the case of perfect competition, and $\theta'_{M,S}$, $\theta'_{M,N}=\theta'_{M,S}<\theta^*$, in the case of monopoly. Also, i) and ii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price in each period is $c(\theta'_{P,S})$. Also, both consumer welfare and social welfare are below the value when $\beta = 1$, but above the value in the naïve consumer case, because there is a distortion but it is smaller than in the naïve consumer case (while producer welfare is unchanged since it equals zero in both cases).
- ii) Given monopoly, behavior is described by ii) of Proposition 2.

Proposition 3 tells us that, in the case of perfect competition, the equilibrium durability level under present bias with sophisticated consumers is below the efficient level, but the distortion is smaller than when consumers are naïve. However, in the case of monopoly, the sophisticated consumer equilibrium is identical to the naïve consumer equilibrium. The logic for the result concerning perfect competition is that sophisticated consumers realize they will exhibit present bias in the future and thus that in the future they will purchase units with inefficiently low durability levels. Anticipating this, they purchase units with a higher durability level than naïve consumers who believe future choices will be efficient. The logic for the monopoly result is that both naïve and sophisticated consumers anticipate that the monopolist will extract all future surplus, so willingness to pay in the current period for units of a given durability level is independent of whether a consumer is sophisticated or naïve. The result is the same durability level for both sophisticated and naïve consumers.

⁷There may be non-stationary Markov Perfect equilibria in the case of sophisticated consumers. We follow O'Donoghue and Rabin (2002) and Acharya et al. (2023) in focusing on stationary Markov Perfect equilibria.

Our final analysis in this subsection concerns a comparison of equilibrium durability choice under perfect competition and monopoly given present-biased consumers. This comparison is important for our discussion of the Phoebus cartel that appears later.

Corollary 1 (to Propositions 2 and 3). Suppose consumers have constant marginal utility for the numeraire good. Then durability choices in the various cases satisfy $\theta_{M,S'} = \theta_{M,N'} < \theta_{P,N'} < \theta_{P,S'} < \theta_M = \theta_P = \theta^*$.

A detailed proof is provided in the Online Appendix. The corollary tells us that the durability distortion due to present bias is larger in the case of monopoly than in the case of perfect competition. The logic is that the monopolist's incentive to distort durability is based on, V, i.e., consumer willingness to pay for durable-good services, while under perfect competition the incentive to distort durability is based on the cost of producing the durable good. Because willingness to pay from a per period standpoint is higher, the monopolist has a higher incentive to distort the durability decision.

In summary, in the case of constant marginal utility for the numeraire good, consumers with time-consistent preferences purchase units with efficient durability given both perfect competition and monopoly, while consumers with present-biased preferences purchase units with inefficiently low built-in durability under both perfect competition and monopoly. In the monopoly case the degree of distortion is independent of whether consumers are naïve or sophisticated. However, in the case of perfect competition, the distortion is larger when consumers are naïve. Also, the degree of distortion is larger in the case of monopoly than under perfect competition.

C) Decreasing Marginal Utility for the Numeraire Good

In this subsection, we explore how results change when marginal utility for the numeraire good is decreasing rather than constant, i.e., f''(.) < 0. To avoid redundancy, in this subsection we mostly discuss results while additional formal statements concerning the decreasing marginal utility case can be found in the online Appendix.

We start the discussion with the case of perfect competition. In that case results are basically unchanged. First, with time consistent preferences, consumers purchase units with the efficient durability level. Second, with present-biased preferences and naïve consumers, consumers purchase units with a durability level that is below the efficient level. Third, with present-biased preferences and sophisticated consumers, the durability level is below the efficient level, but the distortion is smaller.

The basic logic for these results is the same as in the case of constant marginal utility for the numeraire good. Most importantly, because present-biased consumers in every period perceive lower expected utility from future consumption, they purchase units with lower than efficient durability in order to increase current consumption. In addition, sophisticated consumers in each period realize they will behave this way in future periods, but would like to limit the effect reduced durability in

future periods due to present bias has on future expected utility. The result is a smaller distortion of the durability level in the sophisticated consumer case.

Now consider the case of monopoly. In contrast to the case of perfect competition, the qualitative nature of results changes in the case of monopoly. That is, there is no longer a clear ordering of durability levels across the various possibilities for consumer preferences. For example, it is no longer the case that consumers with time-consistent preferences necessarily purchase units with the efficient durability level, i.e., it is possible that time-consistent consumers purchase units with either higher or lower than efficient durability. Also, present-biased consumers do not necessarily consume units of lower durability than the durability purchased by time-consistent consumers.

There is an important part of the parameter space, however, in which the results found in the previous subsection hold in the monopoly case with decreasing marginal utility. This is captured in Proposition 4 below.

Proposition 4. Suppose the one period utility from consuming the durable good equals αV , the cost of producing a unit of durability θ is given by $\alpha c(\theta)$, and the market structure is monopoly. Holding all other parameters fixed, if there is decreasing marginal utility for the numeraire good, as α approaches zero from above, all the results concerning the monopoly case given constant marginal utility for the numeraire good continue to hold.

A detailed proof is provided in the Online Appendix. Proposition 4 tells us that all the results in the monopoly case in the previous subsection continue to hold in the decreasing marginal utility case for the numeraire good when purchasing the durable good has a small effect on the amount of the numeraire good purchased.⁸ First, the durability choice in the case of time-consistent consumers equals the first-best level. Second, durability choice is the same given present-biased consumers who are naïve and present-biased consumers who are sophisticated, where this durability level is below the efficient level. Third, this level is also below the durability level given perfect competition and naïve consumers (as well as sophisticated consumers) who exhibit present bias. In other words, when the durable good costs a small fraction of consumer income, then the decreasing marginal utility case exhibits equilibrium behavior qualitatively identical to the constant marginal utility case.

4 PRESENT BIAS AND NEW-PRODUCT INTRODUCTIONS

In this section we present our model of new-product introductions, where we investigate how present bias affects the speed at which new-product introductions take place. The first subsection presents the model, while in the second we analyze the model both when consumers have time-consistent preferences and when consumers have present-biased preferences. Our main finding is that, in the

⁸We also assume that the cost of producing the durable good is small in order to ensure that $\alpha V > \alpha C$ holds as α approaches zero from above.

case of perfect competition, present bias can cause a new-product introduction to occur earlier than is efficient and similarly earlier than what occurs when consumers have standard preferences.

A) The Model

We consider a three-period model in which there are N identical consumers who are alive all three periods. Consumers derive utility from the consumption of a durable good and the consumption of other goods which are again referred to as the numeraire good. Utility for representative consumer i in period t, $\mu_{i,t}$, is given by equation (3).

(3)
$$\mu_{i,t} = vb_{i,t}Q_{i,t} + x_{i,t}$$

$$s.t. \ w_{i,t} + x_{i,t} = w$$

In equation (3), $b_{i,t} = 1$ ($b_{i,t} = 0$) when the consumer owns (does not own) a unit of the durable good in period t, where the consumer can own a unit by either purchasing a new unit in period t or by owning a unit purchased in the previous period. $Q_{i,t}$ is the quality of the unit owned by the consumer in period t ($Q_{i,t} = 0$ if the consumer does not own a unit) and v is the value consumers place on a unit of quality of the durable good. In the budget-constraint equation, $w_{i,t}$ denotes the consumer's expenditure in period t purchasing the durable good and w is the per period income. Also, in this model we focus solely on the case in which the marginal utility from the consumption of other goods, i.e., the numeraire good, is constant and equal to one rather than allowing for decreasing marginal utility for the consumption of the numeraire good within a period. Results in the decreasing marginal utility case are qualitatively similar, and we focus on the constant marginal utility case in order to simplify the algebra and make the basic logic of our results easier to follow.

In each period t, consumer i chooses behavior consistent with maximizing her perception of expected utility over the remainder of her lifetime. To be precise, in period 1 consumer i maximizes $U_{i,1}$ which is given in equation (4), while in period 2 the consumer maximizes $U_{i,2}$ which is given in equation (5) (in period 3 the consumer maximizes $\mu_{i,3}$ which is given in equation (3)).

(4)
$$U_{i,1} = \mu_{i,1} + \beta \sum_{\tau=3}^{3} \delta^{\tau-1} \mu_{i,t}$$

$$(5) U_{i,2} = \mu_{i,2} + \beta \delta \mu_{i,3}$$

 δ , $0 < \delta < 1$, is the discount factor and, as before, including β means we allow for the possibility that consumers are present biased and are characterized by hyperbolic discounting. Also, as before, $\beta = 1$ means the consumers have time-consistent or standard preferences, while $\beta < 1$ means that

⁹As in the previous model, we do not allow secondhand-market trade, but that has no significant effect on equilibrium behavior given consumers are identical.

consumers have time-inconsistent or present-biased preferences.

We assume there is exogenous technological progress. To be precise, a product introduced and sold in period t is of quality Q_t , where our assumption of exogenous technological progress means $Q_3 > Q_2 > Q_1$. We assume $vQ_1 > c$ which ensures consumers purchase new units in period 1. Since in any period, a consumer only receives utility from consuming a single unit of the durable good, a consumer who purchases a new unit in any period t who already owns a used unit will sell the used unit for scrap. We assume the scrap value for a unit of the durable good is $z, z \ge 0$, i.e., the scrap value is independent of the quality of the used unit.¹⁰

Note that in this model units are perfectly durable. For example, a unit produced in period 1 is of quality Q_1 whether consumed in period 1, period 2, or period 3. We could instead assume that quality declines with the age of the unit, but the qualitative results would be unchanged, although the logic driving the results would be less transparent. Thus, in order to keep the intuition for the results easier to follow, we assume that units are perfectly durable. As in the earlier model, we also assume no borrowing or savings.

We consider both the case of monopoly and the case of perfect competition. In the case of perfect competition firms are identical, where a firm can produce durable units in each period at a constant marginal cost c, c > z, and no fixed cost. The assumption c > z tells us that it is not profitable to produce units of the durable good that are immediately scrapped. Firms are characterized by the same discount factor δ as are consumers. In the case of monopoly, everything is the same except there is a single producer rather than multiple identical producers. Also, our focus in this section is Subgame Perfect Nash equilibrium.

As a final point concerning the setup of the model, because of the three-period nature of the model and the parameter restriction $vQ_1 > c$ which ensures that consumers purchase a durable unit in period 1, equilibrium behavior in the present-bias case is mostly independent of whether consumers are naïve or sophisticated. There are some small differences, specifically concerning the first-period price, and in our formal analysis we focus on the naïve consumer case in order to limit redundancy.

B) Analysis

We begin our analysis by focusing on parameterizations for which it is efficient for a new product to be introduced in either period 2 or period 3, but not both periods. The parameter restriction that ensures this is $v(Q_3-Q_1) > c-z > v(Q_3-Q_2)$. The first part of this expression, $v(Q_3-Q_1) > c-z$, tells us that it is efficient to introduce a new product in period 3 if a new product was not introduced in period 2, while $c-z > v(Q_3-Q_2)$ tells us that it is not efficient to introduce a new product in period 3 if a new product was introduced in period 2. The focus in investigating this part of the

¹⁰Introducing a scrap value in Section 3's model would not change the qualitative nature of the results. We decided not to introduce a scrap value for that model since the applications we discuss in Section 5 of Section 3's model are not characterized by a scrap value.

parameter space is whether the new-product introduction occurs in the efficient period, and also how present bias affects the date and the efficiency of the new-product introduction.

Let us start by considering first-best behavior for these parameterizations. If consumers purchase new units in period 2, then the increase in social welfare over the three periods relative to new units being purchased neither in period 2 nor period 3 is denoted Δ_2 and is given in equation (6).

(6)
$$\Delta_2 = \delta[v(Q_2 - Q_1) + z - c] + \delta^2 v(Q_2 - Q_1)$$

If instead the new product is introduced in period 3, then the increase in welfare, denoted Δ_3 , is given in equation (7).

(7)
$$\Delta_3 = \delta^2 [v(Q_3 - Q_1) + z - c]$$

Equations (6) and (7) yield that, holding the other parameters fixed, there exists a value Q_2^* , $Q_2^* = \left[\frac{\delta(Q_3 - Q_1)}{1 + \delta}\right] + Q_1 + \left[\frac{(1 - \delta)(c - z)}{v(1 + \delta)}\right]$, such that it is efficient to introduce a new product in period 2 when $Q_2 > Q_2^*$ and to introduce a new product in period 3 when $Q_2 < Q_2^*$.

We next consider behavior for these parameterizations when consumers have standard or timeconsistent preferences.

Proposition 5. Suppose $v(Q_3 - Q_1) > c - z > v(Q_3 - Q_2)$. If consumers are time consistent, i.e., $\beta = 1$, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, all consumers then purchase a new durable unit in period 2 and scrap the used unit (purchase a new durable unit in period 3 and scrap the used unit) if $Q_2 > Q_2^*$ ($Q_2 < Q_2^*$). Also, i), ii), and iii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price for a durable good in each period is c and all surplus is received by consumers.
- ii) Given monopoly and $Q_2 > Q_2^*$, the price for a new unit in period 1 equals $(1 + \delta + \delta^2)vQ_1$, while the price for a new unit in period 2 equals $(1 + \delta)v(Q_2 Q_1) + z$. Also, all surplus is received by the monopolist.
- iii) Given monopoly and $Q_2 < Q_2^*$, the price for a new unit in period 1 equals $(1 + \delta + \delta^2)vQ_1$, while the price for a new unit in period 3 equals $v(Q_3 Q_1) + z$. Also, all surplus is received by the monopolist.

Proposition 5 tells us that, when consumers have time-consistent preferences, then the new-product introduction occurs in the efficient period. The logic is as follows. In this model, given time-consistent preferences, consumers receive all the surplus under perfect competition, while the single firm receives all the surplus in the monopoly case. Given time-consistent preferences and perfect competition, since consumers receive all the surplus and current preferences match long-run utility which is used to define efficient behavior, consumers have an incentive to purchase new units

at the efficient date. Also, a similar logic explains why monopoly behavior is efficient with the difference being in that case it is the monopolist that captures all of the surplus.

Why would present bias make a difference? For the parameterizations we are currently focused on, if new units are purchased in period 2, then new units are not purchased in period 3. We start with the case of perfect competition. By purchasing new durable units in period 2, consumers give up the period-3 utility due to the incremental quality associated with consuming a period-3 new unit rather than a period-2 new unit, i.e., $v(Q_3 - Q_2)$. But with present bias, in period 2 consumers undervalue this potential extra period-3 utility. The result is that, in the case of perfect competition, a new product is produced and sold in period 2 more often than is efficient.

Now consider the same set of parameterizations in the case of monopoly and present bias. Due to present bias, consumer willingness to pay for period-3 utility associated with consuming the durable good in that period is higher in period 3 than in period 2. This gives the monopolist an extra incentive to not sell new units in period 2 and instead delay selling new units until period 3. Thus, in contrast to the case of perfect competition, in the monopoly case a new product is produced and sold in period 2 less often than is efficient.

In Proposition 6 we formalize these results.

Proposition 6. Suppose $v(Q_3-Q_1)>c-z>v(Q_3-Q_2)$. If consumers are present biased, i.e., $\beta<1$, and naïve, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, in the case of perfect competition (monopoly) there exists a value $Q_{2,P'}$ ($Q_{2,M'}$), $Q_{2,P'}< Q_2^*$ ($Q_{2,M'}>Q_2^*$), such that all consumers then purchase a new durable unit in period 2 and scrap the used unit if $Q_2>Q_{2,P'}$ ($Q_2>Q_{2,M'}$), while consumers purchase a new durable unit in period 3 and scrap the used unit if $Q_2<Q_{2,P'}$ ($Q_2<Q_{2,M'}$). Also, i), ii), and iii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price for a durable good in each period is c and all surplus is received by consumers, where social welfare and consumer welfare are both below (equal to) the values when $\beta = 1$ if $Q_{2,P'} < Q_2 < Q_2^*$ ($Q_2 < Q_{2,P'}$ or $Q_2 > Q_2^*$).
- ii) Given monopoly and $Q_2 > Q_{2,M'}$, the price for a new unit in period 1 equals $(1+\delta\beta+\delta^2\beta)vQ_1$, while the price for a new unit in period 2 equals $(1+\beta\delta)v(Q_2-Q_1)+z$. Also, the surplus is shared between the consumers and the monopolist, where consumer surplus is higher and monopoly profit is lower than when $\beta = 1$.
- iii) Given monopoly and $Q_2 < Q_{2,M'}$, the price for a new unit in period 1 equals $(1+\delta\beta+\delta^2\beta)vQ_1$, while the price for a new unit in period 3 equals $v(Q_3-Q_1)+z$. Also, the surplus is shared between the consumers and the monopolist, where consumer surplus is higher and monopoly profit is lower than when $\beta = 1$.

In addition to showing that present bias has a different effect on the incentive to distort the date of the new-product introduction depending on whether the market structure is perfect competition or monopoly, it is interesting to note that monopoly does not result in all surplus going to the monopolist (this was also true in Proposition 2). That is, even though the monopolist charges consumers their willingness to pay each period, given present bias, consumers receive positive surplus in equilibrium. This is because, even after correcting for discounting, consumer willingness to pay in any period t for services in a period t', t' > t, is below the value the consumer actually derives from these services when t' actually arrives.

In the next part of the analysis, we focus on a different part of the parameter space. In particular, we now consider parameterizations for which it is never efficient to introduce a new product in period 3, but Q_3 is sufficiently high that it would be efficient to introduce a new product in period 2 if Q_2 is sufficiently close to Q_3 , i.e., $(1+\delta)v(Q_3-Q_1) > c-z > v(Q_3-Q_1)$. Note that the closer δ is to one, i.e., there is little standard discounting, the larger the range of parameterizations that satisfy this condition.

We again start by considering first-best behavior. If consumers purchase new units in period 2, then the increase in social welfare over the three periods relative to new units being purchased neither in period 2 nor period 3 is again denoted Δ_2 and is given in equation (6). Equation (6) yields that, holding other parameters fixed, there exists a value $Q_2^{**} = Q_1 + \left[\frac{c-z}{(1+\delta)v}\right]$, such that it is efficient to introduce a new product in period 2 when $Q_2 > Q_2^{**}$ and not to introduce a new product in period 2 when $Q_2 < Q_2^{**}$.

We now show that standard or time-consistent preferences yield efficient outcomes for these parameterizations.

Proposition 7. Suppose $(1+\delta)v(Q_3-Q_1) > c-z > v(Q_3-Q_1)$. If consumers are time consistent, i.e., $\beta=1$, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, all consumers then purchase a new durable unit in period 2 and scrap the used unit (do not purchase a new durable unit in either period 2 or period 3) if $Q_2 > Q_2^{**}$ ($Q_2 < Q_2^{**}$). Also, i), ii), and iii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price in each period is c and all surplus is received by consumers.
- ii) Given monopoly and $Q_2 > Q_2^{**}$, the price for a new unit in period 1 equals $(1 + \delta + \delta^2)vQ_1$, while the price for a new unit in period 2 equals $(1 + \delta)v(Q_2 Q_1) + z$. Also, all surplus is received by the monopolist.
- iii) Given monopoly and $Q_2 < Q_2^{**}$, the price for a new unit in period 1 equals $(1 + \delta + \delta^2)vQ_1$. Also, all surplus is received by the monopolist.

As before, when consumers have standard preferences, choices concerning when a new product is introduced are efficient. This is because consumers receive all the surplus in the case of perfect competition and thus have an incentive in that case to make efficient new-product purchases. On the other hand, with monopoly the firm receives all the surplus, and thus in that case the firm has an incentive to introduce new products in an efficient fashion.

We now consider this set of parameterizations when consumers are present biased.

Proposition 8. Suppose $(1+\delta)v(Q_3-Q_1)>c-z>v(Q_3-Q_1)$. If consumers are present biased, i.e., $\beta<1$, and naïve, then under both perfect competition and monopoly, all consumers purchase a durable unit in period 1. In addition, holding all other parameters fixed, in the case of perfect competition (monopoly) there exists a value $Q_{2,P''}$ ($Q_{2,M''}$), $Q_{2,P''}>Q_2^{**}$ ($Q_{2,M''}>Q_2^{**}$), such that all consumers purchase a new durable unit in period 2 and scrap the used unit if $Q_2>Q_{2,P''}$ ($Q_2>Q_{2,M''}$), while consumers do not purchase a new durable unit in either period 2 or period 3 if $Q_2<Q_{2,P''}$ ($Q_2<Q_{2,M''}$). Also, i), ii), and iii) further describe equilibrium behavior.

- i) Given perfect competition, the new-unit price in each period is c and all surplus is received by consumers, where social welfare and consumer welfare are both below (equal to) the values when $\beta = 1$ if $Q_2^{**} < Q_2 < Q_{2,P''}$ ($Q_2 < Q_2^{**}$ or $Q_2 > Q_{2,P''}$).
- ii) Given monopoly and $Q_2 > Q_{2,M''}$, the price for a new unit in period 1 equals $(1+\delta\beta+\delta^2\beta)vQ_1$, while the price for a new unit in period 2 equals $(1+\beta\delta)v(Q_2-Q_1)+z$. Also, the surplus is shared between the consumers and the monopolist, where consumer surplus is higher than when $\beta=1$ and monopoly profit is lower than when $\beta=1$.
- iii) Given monopoly and $Q_2 < Q_{2,M''}$, the price for a new unit in period 1 equals $(1+\delta\beta+\delta^2\beta)vQ_1$. Also, the surplus is shared between the consumers and the monopolist, where consumer surplus is higher than when $\beta = 1$ and monopoly profit is lower than when $\beta = 1$.

The proposition tells us that for these parameterizations, given both perfect competition and monopoly, present bias results in an incentive not to introduce a new product in period 2. The logic here stems from the basic characteristic of a present-biased consumer. That is, in each period a present-biased consumer places little weight on utility in future periods. Given that for these parameterizations there is no period-3 new product introduction independent of whether there is a new-product introduction in period 2, the effect of introducing present bias is to reduce the value consumers place in period 2 on owning a higher quality durable unit in period 3. This, in turn, reduces consumer willingness to pay for a new durable unit in period 2. The result is that, under both perfect competition and monopoly, there are parameterizations in which a new product is not sold in period 2 when such sales would be efficient.

In summary, in our three-period model, present bias sometimes results in a new product being introduced earlier than is efficient. In particular, this can arise when there is perfect competition and the choice is whether to purchase a new unit in period 2 or a new unit in period 3. However, in other cases present bias results in later than efficient rather than earlier than efficient introduction of a new product, or no introduction in any period when introduction in period 2 is efficient. For example, for the parameterizations for which perfect competition results in an incentive for new

product introduction earlier than the efficient date, monopoly results in an incentive for later rather than earlier new-product introductions.

5 APPLICATIONS

The light bulb industry is a possible application of our analysis - specifically, the analysis concerning built-in durability in Section 3. In the first subsection we discuss the Phoebus cartel, while in the second we discuss recent light bulb regulation. In the third subsection we discuss the medical device industry which is a possible application of the Section 4 analysis. Note that a discussion of the general history of light bulbs can be found in Brox (2010), a discussion of the Phoebus cartel and its behavior concerning the durability of light bulbs produced can be found in Krajewski (2014), while a discussion of recent US light bulb regulation can be found in Tabuchi (2022).

A) The Phoebus Cartel

In December 1924, the Phoebus cartel, or formally the Phoebus S.A. Compagnie Industrielle pour le Développement de l'Eclairage was created in Geneva, Switzerland. All major light bulb manufacturers in the world, including Germany's Osram, the Netherlands' Philips, and France's Compagnie des Lampes, were its members. General Electric was represented by its British subsidiary, International General Electric, as well as the Overseas Group, which consisted of its subsidiaries in Brazil, China, and Mexico. Other members included Hungary's Tungsram, the United Kingdom's Associated Electrical Industries, and Japan's Tokyo Electric.

The Phoebus cartel exercised its market power through strict quantity controls. The cartel divided the world market into national and regional zones, and assigned a sales quota to each of its member companies. Companies that exceeded their quotas were fined. While the Phoebus cartel did not directly fix prices, the quantity controls allowed it to maintain stable prices over time despite falling manufacturing costs. The strictly enforced quota system ensured that the cartel was not subject to any commitment issue concerning durable goods production and falling prices over time. In other words, consumers would not have anticipated that the cartel would reduce prices over time since the quotas allowed the cartel to avoid such an outcome. Also, the light produced by an incandescent light bulb typically does not dim in a significant way during the lifetime of the bulb. As a result, the services received by a consumer in any period from a new bulb were arguably identical or close to identical to the services from a (working) used bulb of a similar design.

As mentioned earlier, the absence of the commitment issue and no reduction in the quality of the services provided as a bulb ages suggest that arguments concerning reduced built-in durability due to Bulow (1986), Waldman (1996a), and Hendel and Lizzeri (1999a) do not apply in this setting. Rather, existing theory due to Swan (1970,1971) suggests that the cartel should have produced bulbs of the efficient durability level, and also that the formation of the cartel should have had no impact on the durability of the bulbs produced. But, in fact, the cartel significantly reduced

durability over time and the behavior is sometimes described as the first successful implementation of planned obsolescence in modern manufacturing.

Before the creation of the Phoebus cartel, the average lifetime of an incandescent bulb was approximately 1,800 hours, while from 1925 to 1934 after the creation of the cartel the average lifetime decreased steadily to approximately 1,200 hours. This reduction in durability was achieved by the cartel through rigorous research, close monitoring, and strict enforcement. Documents found in the corporate archives in Berlin of the cartel member Osram demonstrate that this reduction in durability was intentional on the part of the cartel, and strictly enforced by the cartel. Over time cartel members modified the filament and adjusted the current or voltage in order to decrease the average lifetime of the bulbs, where the cartel standard was adjusted over time to achieve a steadily decreasing durability. In order to enforce these changes, each factory bound by the cartel agreement was required to send samples of its bulbs to a central testing laboratory in Switzerland, where the bulbs were tested to see whether they met the cartel standards. If a factory's bulbs were found to last shorter or longer than the cartel standard, the factory was required to pay a substantial fine.

There are conflicting views concerning the motivation for the decrease in durability over time enforced by the cartel. There is no evidence that the decreased durability lowered production costs. The cartel members, as well as a British government commissioned study, argued that the motivation for the reduction in durability was to produce a higher quality and brighter bulb.¹¹ But others who have studied the episode, partially basing their conclusions on quotes from top management of a cartel member, argue that independent of any change in quality, the reduction in durability was an important goal of the cartel because very long lifetimes served to reduce profitability.¹²

Suppose that reduced durability was indeed an important motivation for the changes in light bulb production induced by the cartel. Is there a plausible economic theory that would explain the behavior? As argued, lack of commitment was not an important factor in this market, so Bulow's 1986 argument concerning reduced durability employed to avoid time-inconsistency problems concerning production levels does not seem relevant. Similarly, because services from a bulb do not reduce substantially as the bulb ages, the argument found in Waldman (1996a) and Hendel and Lizzeri (1999) concerning reduced durability resulting in more effective price discrimination also does not seem to apply.

In contrast, Section 3's model provides a clear explanation for why the cartel might have had an incentive to reduce the expected lifetime of the bulbs. In that model, because consumers are identical, there is no Coase/Bulow time-inconsistency problem in which a monopolist would want to lower price over time and hurt its own profitability. Further, the utility derived in a period from

¹¹See The Monopolies and Restrictive Practices Commission (1951).

¹²For example, Krajewski (2014) reports that after discovering an instance where some members attempted to secretly introduce a longer lasting bulb, Anton Philips who was head of Philips warned that "after the very strenuous efforts we made to emerge from a period of long lifetimes, it is of the greatest importance that we do not sink back into the same mire... supplying lamps that will have a very prolonged life."

a unit of the durable good does not depend on the good's age as long as the unit is in good working order which matches the cartel's situation. But the analysis shows that, if consumer preferences are present biased, then a monopolist selling a durable good such as light bulbs that represent a small proportion of a consumer's budget would want to produce less durable units than would a competitive industry. In addition, the analysis also predicts that durability would be below efficient levels.

We are not arguing that we have definitive proof that an important part of the cartel's motivation for reducing durability was the reduction itself, as opposed to the sole motivation being an increase in quality. However, our reading of the evidence leads us to favor the reduction in durability being an important part of the motivation. But more importantly, we believe our theoretical analysis provides a solid theoretical foundation for why the cartel may have found it beneficial to reduce the durability of its bulbs, while alternative theories for reduced durability by a monopolist do not provide a clear explanation for such behavior in this case.

B) Light Bulb Regulation

Until relatively recently, most light bulbs, especially those used in homes, were incandescent light bulbs. This technology is typically associated with Thomas Edison who began serious research on the topic in the 1870s, although the basic technology was discovered long before Edison's work. The basic technology of an incandescent light bulb is that a wire filament is heated until it glows. There are many advantages of incandescent light bulbs, but one disadvantage is that they are energy inefficient relative to a number of other light bulb technologies. Specifically, a typical incandescent bulb converts less than five percent of the energy produced into visible light, while other technologies such as fluorescent bulbs and LED bulbs are much more energy efficient.

In the last few decades, the light bulb market has changed in a dramatic fashion. Numerous countries have passed regulations that have disadvantaged the production and use of incandescent light bulbs. These regulations typically do not directly make it illegal to manufacture or purchase incandescent bulbs, but rather the regulations focus on energy efficiency. However, given that incandescent bulbs are not energy efficient, the result of the regulations is that incandescent bulbs have gradually exited the market. For example, in the US the Energy and Independent and Security Act of 2007 was enacted that imposed energy efficiency requirements for many types of bulbs, and more recently the Biden administration reversed Trump administration policies with the result that the sale of most incandescent bulbs were prohibited by the middle of 2023. The important point here is that the time series of market shares makes it clear that incandescent bulbs, despite their energy inefficiency, were not fully exiting the market in the absence of regulation.

As discussed above, light bulbs would seem to fit the type of product analyzed by Swan in his series of influential papers in the 1970s. That is, the service flow from new and used bulbs are similar if not identical, so according to Swan a competitive industry (as well as a monopoly) should produce bulbs with the efficient durability level. But this argument seems inconsistent with the

evidence concerning how regulation in this industry has actually worked.

Consider the cost of using incandescent bulbs today versus the cost of using LED bulbs. The LED bulbs are clearly more expensive in terms of their initial purchase price, but LED bulbs last significantly longer and use less energy. For example, Hutton Power and Light (2019) reports estimates of the cost of using two bulbs of similar brightness - a 60 watt incandescent bulb and a 12 watt LED bulb - employing the average cost of electricity in Virginia in 2019. In the article, the authors calculate that it would cost \$93 in total to use incandescent bulbs to produce 1,000 hours of light per year over a ten-year period, while using LED bulbs to produce the same amount of light would cost less than \$20. That is, despite the fact that the typical LED bulb is more expensive, that the bulb lasts longer and uses less energy makes LED lighting significantly cheaper.

In other words, despite the idea that standard theory due to Swan suggests that in this type of market regulation is not needed to achieve efficiency, regulation aimed at improving energy efficiency seems to have moved the market to a more efficient outcome in terms of product durability. As discussed earlier, the main prior arguments concerning limitations of Swan's argument such as those found in Bulow (1986), Waldman (1996a) and Hendel and Lizzeri (1999a) do not seem to explain this outcome. Alternatively, there were initially concerns that light from LED bulbs and bulbs employing other alternative technologies were not as pleasing/high quality as the light emitted from incandescent bulbs. But those disadvantages seem to have been short lived and the quality of light produced by LED bulbs is now considered similar in quality to that of incandescent bulbs.

The fact, however, that regulation resulted in more efficient product durability is consistent with Section 3's model. In that analysis, the presence of present-biased consumers causes durability to not be efficient even when the market is perfectly competitive. So regulation that forces firms to abandon an inefficient technology can both improve efficiency, as well as increase both consumer and social welfare. Note that we are not arguing that we have definitive proof that the correct explanation for why regulation in the light bulb industry improved efficiency is that consumers are present biased. Perhaps, for example, the fact that the industry was competitive dramatically slowed the innovation process, although the existence of the alternative technologies was well known even prior to the regulatory changes. We do, however, find it of interest that the effect of the regulation in this industry seems consistent with what our theoretical model predicts.

C) The Medical Device Industry

A potential application of our analysis concerning new product introductions in Section 4 is the medical device industry. There are two aspects of this industry that serve to make it an appropriate application of the Section 4 analysis. First, as discussed in more detail below, there is substantial evidence indicating that consumers in healthcare markets exhibit a high degree of present bias. Second, as is also discussed in more detail below, although the medical device industry is of course not perfectly competitive since products are typically not homogeneous, there is significant competition in this industry. So Section 4's analysis of perfect competition and the timing of new product

introductions in the presence of present bias potentially applies.

We start by describing evidence for present bias in healthcare markets. There is a literature that estimates the degree of present bias among consumers in various markets and most of the papers find evidence for present bias, and in many cases the estimated degree of present bias is substantial.¹³ From the standpoint of the discussion in this subsection, what is of particular interest is that a number of the studies which find a high degree of present bias are studies focused on consumer behavior concerning healthcare. Specifically, Abaluck et al (2018) and Bai et al. (2021) assume hyperbolic discounting concerning healthcare outcomes and estimate β to be between 0.3 and 0.4.

As also mentioned above, the other important aspect of the medical device industry is the low market concentration leading to significant competition in the industry. There are numerous major players in the industry such as Abbott, Johnson & Johnson, and GE Healthcare. And overall, market concentration suggests that there is significant competition for the development of new products in this industry. For example, Statista estimates that the 10 leading global manufacturers in this market had an aggregate global market share of less than 40 percent. Of course, in specific product categories market concentration might be higher. But overall this seems to be an industry in which competition for the introduction of new products is substantial.

So what does the Section 4 analysis predict concerning this industry? Clearly, this is an industry where if a new product is not introduced in any period t, then there is the possibility of an introduction in later periods. So, given the substantial evidence for present bias in the industry, the relevant proposition would seem to be Proposition 6 which predicts that substantial competition will result in inefficiently early new product introductions and that these introductions will be characterized by inefficiently low quality levels.

What is of interest is that there is evidence, particularly concerning the quality prediction, that behavior in this market is consistent with the theoretical predictions. One piece of evidence is that many descriptions of behavior in the industry by market participants are consistent with the predictions. For example, Fuhr et al. (2013) states that "Executives also say that the pressure to launch products quickly at low cost tends to reward innovation over quality. Some argue that the increasing complexity of end-user environments and the innovative new features of products are challenging the typical medical device quality approach." It is also the case that there are significant concerns about the quality of new products in the medical device industry, as manifested by a high rate of product recalls in the industry (see Thirumala and Sinha (2011) for evidence and discussion).

Another relevant prediction is that, given present bias, new product introductions should be more frequent in markets characterized by higher levels of competition. We are not familiar with any studies in the medical device industry that looks at that specific relationship. But there is evidence consistent with competition leading to a higher frequency of new product introductions in general (see Dai (2022) and Wang (2023)). To the extent that present bias is present in many

¹³Cheung et al. (2021) conducts a meta-analysis based on 13 estimates of β from nine studies that assume hyperbolic discounting. They estimate a mean value of β equal to 0.66 with a 95 percent confidence interval of (0.51, 0.85).

markets as the findings in Cheung et al. (2021) suggests, the findings in Dai (2022) and Wang (2023) provide further evidence consistent with our theoretical approach.

Of course, there are other potential explanations for the evidence we discuss. However, we do feel that it is of interest that healthcare markets seem to be characterized by a high degree of present bias, and at the same time the medical device industry exhibits behavior concerning product quality consistent with our theoretical predictions concerning how markets characterized by substantial competition should behave.

6 CONCLUSION

In many markets the useful lifetime of a product is below what is easily achievable, where this can occur either because built-in durability is low or because of frequent new-product introductions that make used units obsolete. Previous research that has focused on this observation has mostly employed monopoly/market power models to explain the behavior, but in many instances in which the behavior is observed the market seems competitive. In this paper, we have explored how time-inconsistent/present-biased consumer preferences can lead to durability below efficient levels in both competitive and monopoly models. We show that, given both types of market structures, present-biased consumer preferences can lead to built-in durability below efficient levels. Also, given perfect competition but not a monopoly, present bias can also lead to quicker than efficient new-product introductions that make used units obsolete. One factor that leads to these results is that a present-biased consumer has an incentive to move consumption from the future to the present more than is efficient from a long-run perspective, and this leads consumers to have a reduced willingness to pay the incremental cost associated with higher built-in durability.

There are a number of directions in which the analysis in this paper could be extended. One direction we feel is of particular interest is enriching the analysis of new-product introductions to allow for various important real-world factors such as heterogeneous consumers, secondhand-market trade, and R&D investments. Moving in these directions would introduce issues of price discrimination, time inconsistency, and adverse selection as found in various papers such as Waldman (1993), Fudenberg and Tirole (1998), and Hendel and Lizzeri (1999b). We believe it would be of interest to investigate how the possibility of frequent new-product introductions due to present bias interacts with complications that arise when factors such as secondhand-market trade and R&D investments are incorporated into the analysis.

Another direction of interest is investigating renting or leasing when consumers have present bias. In this paper we assume that consumers purchase the durable good which is a reasonable assumption for many durable goods markets such as the market for light bulbs discussed in the first two applications of Section 5. But in many other durable goods markets renting or leasing is common. There is an extensive literature that explores various roles that renting or leasing can play in durable goods markets such as avoiding time inconsistency, reducing adverse selection,

and creating moral hazard problems (see, for example, Bulow (1982), Henderson and Ioannides (1983), and Johnson and Waldman (2003)). We feel that in some cases leasing or renting may play an important role in responding to incentives created by present bias. We thus believe that investigating interactions between present bias and renting/leasing would be an interesting direction for future research.

Finally, in our analysis we have abstracted away from borrowing and saving. Since borrowing and purchasing cheaper goods of limited durability are alternative ways that consumers can move consumption from the future to the present, the two behaviors should be substitutes in settings characterized by present-biased consumers. Investigating this substitutability is an additional topic of interest for future research.

7 REFERENCES

Abaluck, J., J. Gruber, and A. Swanson. 2018. "Prescription Drug use Under Medicare Part D: A Linear Model of Nonlinear Budget Sets." Journal of Public Economics, 164: 106-138.

Acharya, Subas, David Jimenex-Gomez, Dmitri Rachinskii, and Alejandre Rivera. 2023. "Present Bias and the Value of Sophistication." Mimeo, University of Texas at Dallas, Mathematics Department.

Anderson, Simon and Victor Ginsburgh. 1994. "Price Discrimination via Second-Hand Markets." European Economic Review, 38(1): 23-44.

Bai, L., B. Handel, E. Miguel, and G. Rao (2021). "Self-Control and Demand for Preventive Health: Evidence from Hypertension in India." Review of Economics and Statistics, 103(5): 835-856.

Bar-Gill, Oren and Andrew Hayashi. 2021. "Present Bias and Debt-Financed Durable Goods." Discussion Paper No. 1055, Harvard Law School.

Brox, Jane. 2010. Brilliant: The Evolution of Artificial Light. Houghton Mifflin Harcourt, New York.

Bulow, Jeremy. 1982. "Durable-Goods Monopolists." Journal of Political Economy, 90(2): 314-332.

Bulow, Jeremy. 1986. "An Economic Theory of Planned Obsolescence." Quarterly Journal of Economics, 101(4): 729-749.

Cheung, Stephen L., Agnieszka Tymula, and Xueting Wang. 2021. "Quasi-Hyperbolic Present Bias: A Meta-Analysis." Life Course Centre Working Paper No. 2021-15.

Choi, Jay Pil. 1994. "Network Externality, Compatibility Choice, and Planned Obsolescence." Journal of Industrial Economics, 42(2): 167-182.

Coase, Ronald H. 1972. "Durability and Monopoly." Journal of Law and Economics, 15(1): 143-149.

Dai, Ruiting. 2022. "Product Market Competition and Strategic New Product Releases. Yale Graduate School of Arts and Sciences Dissertations.

Fudenberg, Drew and Jean Tirole. 1998. "Upgrades, Trade-ins, and Buy-backs." Rand Journal of Economics, 29(2): 235-258.

Fuhr, Ted, Katy George, and Janice Pai. 2013. "The Business Case for Medical Device Quality." McKinsey Center for Government.

Grout, Paul A. and In-Uck Park. 2005. "Competitive Planned Obsolescence." Rand Journal of Economics, 36(3): 596-612.

Hendel, Igal and Alessandro Lizzeri. 1999a. "Interfering with Secondary Markets." Rand Journal of Economics, 30(1): 1-21.

Hendel, Igal and Alessandro Lizzeri. 1999b. "Adverse Selection in Durable Goods Markets." American Economic Review, 89(5): 1097-1115.

Henderson, J.V. and Y.M. Ioannides. 1983. "A Model of Housing Tenure Choice." American Economic Review, 73(1): 98-113.

Hutton Power & Light. 2019. "LED vs. Incandescent Lighting: A Cost Comparison." Blog Post.

Johnson, Justin P. and Michael Waldman. 2003. "Leasing, Lemons, and Buybacks." Rand Journal of Economics, 34(2): 247-265.

Kang, Jingoo and Minwook Kang. 2022. "Durable Goods as Commitment Devices Under Quasi-Hyperbolic Discounting." Journal of Mathematical Economics, 99: Article 102561.

Kleiman, E. and T. Ophir. 1966. "The Durability of Durable Goods." Review of Economic Studies, 33(2): 165-178.

Krajewski, Markus. 2014. "The Great Lightbulb Conspiracy." IEEE Spectrum, 51(10): 56-61.

Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting." Quarterly Journal of Economics, 112(2): 443-477.

Levhari, David and T. Srinivasan. 1969. "Durability of Consumption Goods: Competition versus Monopoly." American Economic Review, 59(1): 102-107.

Loewenstein, George and Drazen Prelec. 1992. "Anomolies in Intertemporal Choice: Evidence and an Interpretation." Quarterly Journal of Economics, 107(2): 573-597.

Meier, Stephan and Charles Sprenger. 2010. "Present-Biased Preferences and Credit Card Borrowing." American Economic Journal: Applied Economics, 2(1): 193-210.

Mussa, Michael and Sherwin Rosen. 1978. "Monopoly and Product Quality." Journal of Economic Theory, 18(2): 301-317.

Nocke, Volker and Martin Peitz, 2003. "Hyperbolic Discounting and Secondary Markets." Games and Economic Behavior, 44(1): 77-97.

O'Donoghue, Ted and Matthew Rabin. 1999. "Doing it Now or Later." American Economic Review, 89(1): 103-124.

O'Donoghue, Ted and Matthew Rabin. 2002. "Addiction and Present-Biased Preferences." Working Paper No. E02-312, UC Berkeley, Department of Economics.

O'Donoghue, Ted and Matthew Rabin. 2015. "Present Bias: Lessons Learned and to be Learned." American Economic Review, 105(5): 273-279.

Pesendorfer, Wolfgang. 1995. "Design Innovation and Fashion Cycles." American Economic Review, 85(4): 771-792.

Schmalensee, Richard. 1970. "Regulation and the Durability of Goods." Bell Journal of Economics and Management Science, 1(1): 54-64.

Schmalensee, Richard. 1979. "Market Structure, Durability, and Quality: A Selective Survey." Economic Inquiry, 17(2): 177-196.

Sieper, E. and Peter Swan. 1973. "Monopoly and Competition in the Market for Durable Goods." Review of Economic Studies, 40(3): 333-351.

Strotz, R.H. 1955. "Myopia and Inconsistency in Dynamic Utility Maximization." Review of

Economic Studies, 23(3): 165-180.

Swan, Peter L. 1970. "Durability of Consumption Goods." American Economic Review, 60(5): 884-894.

Swan, Peter L. 1971. "The Durability of Goods and Regulation of Monopoly." Bell Journal of Economics and Management Science, 2(1): 347-357.

Tabuchi, Hiroko. 2022. "New Rules Will End the Century-Long Run of Classic Light Bulbs." New York Times, April 26, 2022.

Thiumalai, Sriram and Kingshuk Sinha. (2011). "Product Recalls in the Medical Device Industry: An Empirical Exploration of the Sources and Financial Consequences." Management Science, 57(2): 376-392.

The Monopolies and Restrictive Practices Commission. 1951. Report on the Supply of Electric Lamps. Her Majesty's Stationary Office, London.

Waldman, Michael. 1993. "A New Perspective on Planned Obsolescence." Quarterly Journal of Economics, 108(1): 273-283.

Waldman, Michael. 1996a. "Durable Goods Pricing When Quality Matters." Journal of Business, 69(4): 489-510.

Waldman, Michael. 1996b. "Planned Obsolescence and the R&D Decision." Rand Journal of Economics, 27(3): 583-595.

Waldman, Michael. 2003. "Durable Goods Theory for Real World Markets." Journal of Economic Perspectives, 17(1): 131-154.

Wang, Peichun. 2023. "Product Portfolio Choices with Product Life Cycles." Working Paper.

8 APPENDIX A

8.1 Proof of Proposition 1

The First Best

We first look at the maximization problem of a social planner who maximizes the total welfare by choosing a durability level. The welfare is measured in a time-consistent way. Let $T(\theta)$ stand for total welfare when the durability is θ .

(1)
$$T(\theta) = \max_{\theta} \frac{V}{1-\delta} + f(w - c(\theta)) + \frac{(1-\theta)\delta f(w - c(\theta))}{1-\delta} + \frac{\theta \delta f(w)}{1-\delta}$$
$$T(\theta) = \max_{\theta} v + Zw - (1-\delta\theta)c(\theta)Z$$

The first three terms $\frac{V}{1-\delta} + Z(w-c(\theta))$ stand for the first period utility of purchasing a durable good with durability θ . A fraction of the income $c(\theta)$ was used for producing the durable good and thus the consumption of the numeraire good decreased. The third term means that every period, there is a probability $1-\theta$ that the durable product breaks, and has to be produced again. The last term means with probability θ the durable good lasts for one more period so that the consumer can use all the income w to purchase the numeraire goods. The same welfare expression can be derived with recursive formulation, but for simplicity, we just provide the resulting form here.

As we will see later, this maximization problem is exactly the same as the maximization problem for time-consistent consumers. So the First Best is achieved by the competitive market with timeconsistent consumers.

While the first best FOC is the following:

(2)
$$[FOC] \theta^*: 0 = \frac{\delta}{1 - \delta} f(w) - \frac{\delta}{1 - \delta} f(w - c(\theta^*)) - \frac{1 - \delta \theta^*}{1 - \delta} f'(w - c(\theta^*)) c'(\theta^*)$$
$$[FOC] \theta^*: 0 = -\frac{\delta}{1 - \delta \theta^*} c(\theta^*) - c'(\theta^*)$$

Competitive Market

We solve the competitive market equilibrium with infinite periods and no saving or borrowing. Let $L \in \{0,1\}$ be a state variable that denotes whether the consumer owns a durable good at the beginning of the period. Let $v_1(\theta, L=1)$ denote the expected utility of a time-consistent consumer entering a period with a working durable good of durability θ . Denote $v_1(L=0)$ as the anticipated value for the future period of not having a durable good. In future sections, $v_1(\cdot)$ is also the anticipated value for the future period of a present-biased naïve consumer because such a consumer anticipates him or herself to be time-consistent in the future. Sophisticated consumers, on the other hand, have different anticipated values. We will discuss sophisticated consumers in

the proof of future propositions.

In a competitive market, due to the zero-profit condition and constant marginal cost (with respect to the amount of production), we can conclude that the price of the product always equals the marginal cost of production for any given durability level θ .

$$p(\theta) = c(\theta)$$

Substituting the price with the marginal cost, we can solve the time-consistent part of the value function $v_1(\cdot)$. It follows the Bellman equations below:

$$v_{1}(L=0) = \max_{\theta_{1}} \left[f(w - c(\theta_{1})) \right] + V + \delta \theta_{1} v_{1}(\theta_{1}, L=1) + \delta (1 - \theta_{1}) v_{1}(L=0),$$

$$(3) \qquad v_{1}(\theta_{1}, L=1) = f(w) + V + \delta \theta_{1} v_{1}(\theta_{1}, L=1) + \delta (1 - \theta_{1}) v_{1}(L=0),$$

$$v_{1}(\theta_{1}, L=1) = \frac{1}{1 - \delta \theta_{1}} \left[f(w) + V + \delta (1 - \theta_{1}) v_{1}(L=0) \right].$$

Here, θ_1 is just a placeholder variable that can take any value from 0 to 1 in the second and third value function. The first value function only holds when taking the max over $\theta_1 \in [0, 1]$, while the latter two equations always hold by definition.

Notice that there is no maximization operator for $v_1(\theta_1, L=1)$. Because whenever the consumer has a working durable good, she does not have a choice variable for that period. She just spends all her income on consumption goods. (The same logic applies when we solve the present bias consumers' decisions.)

Thus, we can move $v_1(\cdot)$ terms to the left hand side and get the following:

$$(1 - \delta(1 - \theta_1))v_1(L = 0) - \delta\theta_1v_1(\theta_1, L = 1) = \max_{\theta_1} [f(w - c(\theta_1)) + V]$$

Then, we can express $v_1(\theta_1, L=1)$ as a function of $v_1(L=0)$:

$$(1 - \delta(1 - \theta_1))v_1(L = 0) - \delta\theta_1 \frac{1}{1 - \delta\theta_1} [f(w) + V + \delta(1 - \theta_1)v_1(L = 0)] = \max_{\theta_1} [f(w - c(\theta_1)) + V]$$

Thus we can solve for $v_1(L=0)$ as the following:

$$v_1(L=0) = \max_{\theta_1} \left[\frac{V}{1-\delta} + \frac{\delta}{1-\delta} \theta_1 f(w) + \frac{1-\delta \theta_1}{1-\delta} f(w - c(\theta_1)) \right]$$

Let θ_P be the optimal solution for the time-consistent part, which coincides with the first-best durability in a competitive market. We can then solve the system of equations and derive the following expression for the value functions:

$$(4) v_1(L=0) = \max_{\theta_P} \left[\frac{V}{1-\delta} + \frac{\delta}{1-\delta} \theta_P f(w) + \frac{1-\delta\theta_P}{1-\delta} f(w - c(\theta_P)) \right],$$

$$v_1(\theta_P, L=1) = \frac{V}{1-\delta} + \left(\frac{\delta\theta_P}{1-\delta} + 1 \right) f(w) + \frac{\delta - \delta\theta_P}{1-\delta} f(w - c(\theta_P)).$$

Where θ_P is the expected optimal choice of durable good durability. It is obtained by solving the following first-order condition (FOC), by collecting all terms that contain $v_1(\cdot)$ to the left-hand side:

(5)
$$[FOC] \theta_P : \frac{\delta}{1-\delta} f(w) - \frac{\delta}{1-\delta} f(w - c(\theta_P)) - \frac{1-\delta\theta_P}{1-\delta} f'(w - c(\theta_P)) c'(\theta_P) = 0$$

Then, we can plug in f(w) = Zw, and the maximization problem becomes the following one:

(6)
$$[FOC] \theta_P : \delta c(\theta_P) - (1 - \delta \theta_P)c'(\theta_P) = 0$$

This FOC is exactly the same as the first-best solution. Therefore, for time-consistent consumers, a competitive market achieves the first best. Thus, we get the following lemma:

Lemma 1. In a competitive market with time-consistent consumers, the market durability equals the first best $\theta_P = \theta^*$.

For future proof, $v_1(L=0)$ is a commonly used value. It means the total utility of a time-consistent consumer in a first-best market.

(7)
$$v_1(L=0) = \frac{V}{1-\delta} + \frac{\delta}{1-\delta} \theta^* f(w) + \frac{1-\delta\theta^*}{1-\delta} f(w - c(\theta^*))$$

Monopoly Market

In this section, we solve the monopoly market for time-consistent consumers. We start the proof with $f(\cdot)$ inside, allowing us to apply the first order conditions for later use.

If the consumers are time consistent, then the expected utility of buying a durable good with durability θ and paying price p at the current period is denoted as $U_1(\theta, p)$. The utility of owning a working durable good (no need to pay) with durability θ is $U_1(\theta, 0)$, where p = 0 if the consumer owns a working durable good and there is no need to pay:

(8)
$$U_1(\theta, p) = f(w - p) + V + \delta\theta U_1(\theta, 0) + \delta(1 - \theta)U_1(\theta, p),$$
$$U_1(\theta, 0) = f(w) + V + \delta\theta U_1(\theta, 0) + \delta(1 - \theta)U_1(\theta, p).$$

We can solve them as:

$$U_{1}(\theta, p) - f(w - p) + f(w) = U_{1}(\theta, 0),$$

$$(9) \qquad U_{1}(\theta, p) = f(w - p) + V + \delta\theta U(\theta, p) - \delta\theta f(w - p) + \delta\theta f(w) + \delta(1 - \theta)U_{1}(\theta, p),$$

$$U_{1}(\theta, p) = \frac{1}{1 - \delta} [f(w - p) + V + \delta\theta f(w) - \delta\theta f(w - p)].$$

While if the consumer never buys the product, the expected utility is:

(10)
$$U_1(\text{no}) = \frac{f(w)}{1 - \delta}$$

Thus, the willingness to pay for the durable good is the largest p, such that:

(11)
$$U_{1}(\theta, p) \geq U_{1}(\text{no}),$$

$$\frac{1}{1 - \delta} \left[f(w - p) + V + \delta \theta f(w) - \delta \theta f(w - p) \right] \geq \frac{f(w)}{1 - \delta},$$

$$f(w - p) \geq f(w) - \frac{V}{1 - \delta \theta}.$$

Since all the consumers are homogeneous, if the price is larger than the critical value, then the demand will drop to zero. If the price is less than or equal to the critical price, the demand will be one unit per person. Thus, for each durability level, the monopolist always chooses the price equal to the critical price, and we do not need to add price into the maximization problem. Instead, we can solve for p as an explicit function of θ .

(12)
$$p_{TC}(\theta) = w - f^{-1} \left(f(w) - \frac{V}{1 - \delta \theta} \right),$$
$$p'_{TC}(\theta) = \frac{V \delta}{f' \left(f^{-1} \left(f(w) - \frac{V}{1 - \delta \theta} \right) \right) (1 - \delta \theta)^2}.$$

By definition of the critical prices, we have the following utility equality:

(13)
$$U_1(\theta, p_{TC}(\theta)) \ge U_1(\text{no}) = \frac{f(w)}{1 - \delta}.$$

By taking the derivative, we can conclude that $p(\theta)$ is increasing and concave. Then we can calculate the profit of the firm as:

(14)
$$\pi(\theta) = \max_{\theta} \left[p_{TC}(\theta) - c(\theta) + \frac{\delta(1-\theta)(p_{TC}(\theta) - c(\theta))}{1-\delta} \right]$$
$$= \max_{\theta} \left[(1 + \frac{\delta(1-\theta)}{1-\delta})(p_{TC}(\theta) - c(\theta)) \right].$$

Then, we have the FOC to solve for the optimal θ :

(15)
$$[FOC] \theta_M : 0 = -\frac{\delta}{1 - \delta} (p_{TC}(\theta_M) - c(\theta_M)) + (1 + \frac{\delta(1 - \theta_M)}{1 - \delta}) (p'_{TC}(\theta_M) - c'(\theta_M)),$$
$$0 = -\frac{\delta}{1 - \delta\theta_M} (p_{TC}(\theta_M) - c(\theta_M)) + p'_{TC}(\theta_M) - c'(\theta_M).$$

We can further plug in all the functional forms and get the resulting FOC

$$[FOC] \theta_M : 0 = \frac{\delta}{1 - \delta\theta} \left(-w + f^{-1} \left(f(w) - \frac{V}{1 - \delta\theta} \right) + c(\theta) \right) + \frac{V\delta}{f' \left(f^{-1} \left(f(w) - \frac{V}{1 - \delta\theta} \right) \right) (1 - \delta\theta)^2} - c'(\theta).$$

Then, we can substitute f(w) = Zw into this FOC and then we have:

(17)
$$[FOC] \theta_M : 0 = \frac{\delta}{1 - \delta\theta} \left(\frac{-V}{Z(1 - \delta\theta)} + c(\theta) \right) + \frac{V\delta}{Z(1 - \delta\theta)^2} - c'(\theta).$$

$$[FOC] \theta_M : 0 = \frac{\delta}{1 - \delta\theta} c(\theta) - c'(\theta).$$

Which is exactly the same as the FOC for the competitive market and the first best. Thus, we have proved that: in both perfect competition and monopoly all consumers consume a durable unit in each period and firms produce units of durability θ^* . And this completes the proof of Proposition 1.

8.2 Proof of Proposition 2

To prove this proposition, we solve the competitive/monopoly market durability for naïve present biased consumers. Some of the results in this proof can be demonstrated with a weakly concave f(.) function that satisfies the following assumption. The results of this section depend on the following lemma, so I will show it here.

Lemma 2. When $f(\cdot)$ is weakly concave and weakly increasing, and $c(\cdot)$ is strictly convex and strictly increases, $f(w - c(\theta))$ must be strictly concave in θ .

The detailed proof is provided in the online appendix due to page limitation.

Competitive Market with Naïve Consumers

In this section, we derive the first-order conditions for a competitive market consisting of naïve Consumers. We start the proof by solving the value function of time time consistent part of the preference, which is the naïve consumer's expectation of their future purchasing behavior. Then, we plug the corresponding expected value into the current period's value function and solve the current period decision.

Due to the page limit, the details are provided in the online appendix. Here, we only show the resulting first-order conditions.

Let $\theta_{P,N'}$ denotes the durability of competitive market (P) with naïve consumers (N').

[FOC] Comp naïve:
$$0 = -c'(\theta_{P,N'}) + \beta \left[\frac{\delta(1 - \delta\theta^*)c(\theta^*)}{(1 - \delta\theta_{P,N'})^2} \right]$$
[FOC] First Best:
$$0 = -c'(\theta^*) + \frac{\delta c(\theta^*)}{1 - \theta^* \delta}$$

By comparing the two first-order conditions, we can see that $\theta_{P,N'}$ is weakly smaller than θ^* because $c(\cdot)$ is a convex function.

Monopoly Market naïve Consumers

The value functions for time-consistent consumers have been solved in the previous proof. So, in this section, we will keep the same notation and continue with the first-period decision of the naïve consumers. Because of the β discount factor, the willingness to pay for the durable good should be lower in each period. So, naturally, the monopolist will choose less durable goods at a lower price.

In this section, we consider a Markov-stationary equilibrium. The major assumption is that the seller always offers a durable good with the same price and durability. Thus, the consumers also anticipate the same product will always be offered in future periods at the same price.

The detailed proof is provided in the online appendix. We only show the final first-order conditions here. Plug in f(w) = Zw, where Z represents the slope. The above FOC becomes the following one. We can thus directly compare it with the FOC of the first best:

(19)
$$[FOC] \ Monopoly: \ 0 = -c'(\theta_{(M,N')}) + \frac{\delta c(\theta_{(M,N')})}{(1 - \delta \theta_{(M,N')})} + \frac{V\delta(\beta - 1)}{(Z(1 - \beta \delta \theta_{(M,N')})^2)}$$

$$[FOC] \ First \ Best: \ 0 = -c'(\theta^*) + \frac{\delta c(\theta^*)}{(1 - \theta^* \delta)}$$

In the linear model, the monopolist chooses a less durable goods for the present biased consumer compared to the time-consistent consumer (This is not necessarily true in under all utility and functions). This is because $\frac{V\delta(\beta-1)}{(1-\beta\delta\theta)^2}$ must be negative, if we plug the θ^* into the FOC of $\theta_{(M,N')}$ then the latter FOC becomes negative. So we can conclude $\theta_{(M,N')} < \theta^*$. When the consumers are time-consistent, then the last term becomes zero since $(\beta-1)=0$ in the numerator. So the time consistent solution coincides with the first best.

Welfare Analysis

Competitive case In a competitive market, with time-consistent consumers, all the surplus goes to the consumers because of constant marginal cost and zero profit conditions. When the consumers are present-biased and naïve, consumer welfare decreases since they do not choose the first-best durability. On the other hand, the profit for the seller is always zero by definition of the competitive market.

Monopoly case Higher consumer surplus is measured by a long-run hypothetical time-consistent preference. In a monopoly market, the proof of higher surplus for present-biased consumers compared to time-consistent consumers is straightforward. In time-consistent cause, consumer surplus is zero. When consumers are present-biased, the monopolist always charges a lower price for all θ . Thus, consumer surplus must be weakly higher than zero. Otherwise, they would not buy the durable product.

The monopoly profit is lower when the consumer is present-biased because the seller does not choose the first best durability, so the total surplus is smaller than the time-consistent case. Furthermore, the monopolist always charges a lower price for all θ for present-biased consumers, it also receives a smaller share of the total surplus. Thus, the profit of the seller will be smaller.

And this completes the proof of Proposition 2.

8.3 Proof of Proposition 3

In the previous proposition, we solved the monopoly market in the previous proof with naïve consumers, here we first show that a monopoly market with sophisticated consumers will be the same. In the previous model, we assume that naïve consumers believe that the monopoly seller will choose the same durability for all future periods because we focus on a stationary equilibrium. Thus, such a belief is exactly the same as the sophisticated consumers because both types of consumers anticipate the same durable goods will be offered in all future periods. Consequently, they have the same expectation on the continuation payoff. So we can conclude that the equilibrium durability with sophisticated consumers $\theta_{(M,S')}$ is the same as that of naïve consumers $\theta_{(M,N')}$.

Next, we move on to solve sophisticated consumers in the competitive market. naïve consumers have false anticipation of their future purchasing decisions. Sophisticated consumers, on the other hand, always correctly anticipate their own future decisions. By assuming stationary equilibrium, sophisticated consumers believe that they will continue to purchase the same durability products.

The details of the derivation is provided in online appendix.

Thus, we have the FOC of sophisticated consumers as the following. Notice that the sophisticated consumer makes current purchase decision given the anticipation of the future as fixed. Let $\theta_{P,S'}$ be the durability choice of sophisticated consumers in a stationary equilibrium. So the derivative of $U_1(\theta, c(\theta))$ with respect to the current period choice variable is zero.

$$[FOC]\theta_{P,S'}: 0 = -f'(w - c(\theta))c'(\theta) + \frac{\beta\delta}{(1-\delta)}[f(w) - f(w - c(\theta)) - (1-\theta)f'(w - c(\theta))(-c'(\theta))]$$

So, obviously, $\theta_{P,S'} < \theta^*$, because of lemma 1.

Then, we compare the naive and sophisticated consumer's FOC's

$$[FOC]\theta_{P,S'}: 0 = -f'(w - c(\theta))c'(\theta^*) + \beta\delta \left[\frac{f(w) + V + (1 - \theta)\delta U_1(\theta, c(\theta))}{(1 - \delta\theta)^2} - \frac{U_1(\theta, c(\theta))}{1 - \theta\delta} \right]$$

$$[FOC]\theta_{P,N'}: 0 = -f'(w - c(\theta))c'(\theta^*) + \beta\delta \left[\frac{f(w) + V + (1 - \theta)\delta c(\theta)}{(1 - \delta\theta)^2} - \frac{v_1(L = 0)}{1 - \theta\delta} \right]$$

Here $v_1(L=0)$ a naive consumer's expected value in the future period if he does not have a working light bulb.

 $U_1(\theta, c(\theta))$ denotes a Sophisticated consumer's expected value in the future period of buying a durable good with durability θ and paying price $c(\theta)$ at the current period.

In the first FOC, the combined coefficient in front of $U_1(\theta, c(\theta))$ is negative. Also, we know that $U_1(\theta, c(\theta)) < v_1(L=0)$, sophisticated consumers believe that they will not buy the optimal product in the future. Thus, if we plug $\theta_{P,N'}$ into the FOC of the sophisticated consumer, we will see that the FOC of the sophisticated consumer is strictly positive. Thus, we establish the following result: In a competitive market, $\theta_{P,N'} \leq \theta_{P,S'} \leq \theta^*$

We complete the proof of Proposition 3 for all weakly concave utility functions such that $f''(\theta) \leq 0$.

9 APPENDIX B

PROOF OF PROPOSITION 5 Given the parameter restriction $vQ_1 > c$, in equilibrium all consumers purchase a new unit in period 1. We first consider the case of perfect competition. The zero profit constraint associated with perfect competition means that the new unit price in each period is c.

Consider consumer i. Suppose the consumer did not purchase a new unit in period 2. Given $v(Q_3 - Q_1) > c - z$, the consumer will purchase a new unit in period 3. Thus, at the beginning of period 2 the consumer anticipates that if she does not purchase a new unit in period 2, then her discounted utility over periods 2 and 3 would be equal to $vQ_1 + \delta(vQ_3 - (c - z))$.

Suppose the consumer did purchase a new unit in period 2. Given $v(Q_3 - Q_2) < (c - z)$, the consumer will not purchase a new unit in period 3. Thus, at the beginning of period 2 the consumer anticipates that if she purchases a new unit in period 2, then her discounted utility over periods 2 and 3 would be equal to $vQ_2 - (c - z) + \delta vQ_2$.

Setting the two expressions equal to each other and rearranging yields that the consumer is indifferent between purchasing and not purchasing a new unit in period 2 when $Q_2 = Q_2^*$. Note that the expected utility over periods 2 and 3 when the consumer does not purchase a new unit in period 2 is independent of Q_2 , while expected utility over periods 2 and 3 when the consumer purchases a new unit in period 2 increases with Q_2 . Thus, the consumer purchases a new unit in period 2 if $Q_2 > Q_2^*$, while the consumer instead purchases a new unit in period 3 if $Q_2 < Q_2^*$. Also, since the new-unit price is c in each period, all surplus is received by consumers. This completes

the proof of all the statements in the proposition concerning perfect competition.

We now consider the monopoly case. In any period in which the monopolist sells a unit to a consumer, the monopolist will charge the consumer's willingness to pay which means all the surplus is received by the monopolist. Given this, suppose consumers did not purchase a new unit in period 2. Given $v(Q_3 - Q_1) > c - z$, the monopolist will sell new units to consumers at the price $v(Q_3 - Q_1) + z$ which is the consumer's willingness to pay. So the monopolist's discounted profit over periods 2 and 3 equals $\delta N[v(Q_3 - Q_1) - (c - z)]$.

Suppose instead consumers did purchase new units in period 2. Given $v(Q_3 - Q_2) < c - z$, the monopolist will not sell new units in period 3. Thus, since consumers at the beginning of period 2 will anticipate this behavior, their maximum willingness to pay for new units at the beginning of period 2 is $v(Q_2 - Q_1) + z + \delta v(Q_2 - Q_1)$ which is the price. So the monopolist's discounted profit over periods 2 and 3 equals $N[(1 + \delta)v(Q_2 - Q_1) - (c - z)]$.

Setting the two expressions equal to each other and rearranging yields that the monopolist is indifferent between selling and not selling new units in period 2 when $Q_2 = Q_2^*$. Note that discounted monopoly profit over periods 2 and 3 when the monopolist does not sell new units in period 2 is independent of Q_2 , while discounted profit over periods 2 and 3 when the monopolist sells new units in period 2 increases with Q_2 . Thus, the monopolist sells new units in period 2 if $Q_2 > Q_2^*$, while the monopolist sells new units in period 3 if $Q_2 < Q_2^*$. This completes the proof of all the statements in the proposition concerning monopoly.

PROOF OF PROPOSITION 6 Given the parameter restriction $vQ_1 > c$, in equilibrium all consumers purchase a new unit in period 1. We first consider the case of perfect competition. The zero profit constraint associated with perfect competition means that the new-unit price in each period is c.

Consider consumer i. Suppose the consumer did not purchase a new unit in period 2. Given $v(Q_3 - Q_1) > c - z$, the consumer will purchase a new unit in period 3. Thus, at the beginning of period 2 the consumer anticipates that if she does not purchase a new unit in period 2, then her perceived discounted utility over periods 2 and 3 would be equal to $vQ_1 + \beta\delta(vQ_3 - (c-z))$. Suppose the consumer did purchase a new unit in period 2. Given $v(Q_3 - Q_2) < c - z$, the consumer will not purchase a new unit in period 3. Thus, at the beginning of period 2 the consumer anticipates that if she purchases a new unit in period 2, then her perceived discounted utility over periods 2 and 3 would be equal to $vQ_2 - (c-z) + \beta\delta vQ_2$.

We now consider the monopoly case. In any period in which the monopolist sells new units to consumers, the monopolist will charge consumer willingness to pay. Suppose consumers did not purchase a new unit in period 2. Given $v(Q_3 - Q_1) > c - z$, the monopolist will sell new units to consumers at the price $v(Q_3 - Q_1)$ which is the consumers' willingness to pay. So the monopolist's discounted profit over periods 2 and 3 equals $\delta N[v(Q_3 - Q_1) - (c - z)]$.

Suppose instead consumers did purchase new units in period 2. Given $v(Q_3 - Q_2) < c - z$, the

monopolist will not sell new units in period 3. Thus, since consumers at the beginning of period 2 will anticipate this behavior, their maximum willingness to pay for new units at the beginning of period 2 is $v(Q_2 - Q_1) + z + \beta \delta v(Q_2 - Q_1)$ which is the price. So the monopolist's discounted profit over periods 2 and 3 equals $N[(1 + \beta \delta)v(Q_2 - Q_1) - (c - z)]$.

Setting the two expressions equal to each other and rearranging yields that the monopolist is indifferent between selling and not selling new units in period 2 when $Q_2 = Q_2$, $M' = Q_1 + [\delta(Q_3 - Q_1)/(1+\beta\delta)] + [(1-\delta)(c-z)/(1+\beta\delta)v] > Q_2^*$. Note that the expected utility over periods 2 and 3 when the consumer does not purchase a new unit in period 2 is independent of Q_2 , while the expected utility over periods 2 and 3 when the consumer purchases a new unit in period 2 increases with Q_2 . Thus, the consumer purchases a new unit in period 2 if $Q_2 > Q_{2,M'}$, while the consumer instead purchase a new unit in period 3 if $Q_2 < Q_{2,M'}$.

Consider the cases $Q_2 < Q_2^*$ and $Q_2 > Q_{2,M'}$. The period 2 and period 3 prices for a new unit are given above. In the case $Q_2 > Q_{2,M'}$, a naïve consumer correctly believes that she will purchase a new unit in period 2 and that the monopolist will extract all the surplus. Thus, her period-1 willingness to pay for a new unit is $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. In the case $Q_2 < Q_2^*$, the naïve consumer correctly believes that she will purchase a new unit in period 3 and the monopolist will extract all the surplus. Thus, her period-1 willingness to pay for a new unit is again $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. Note that these prices are less than in the case $\beta = 1$ which means surplus is shared and monopoly profit is lower than in the case $\beta = 1$.

Finally, consider the case $Q_2^* < Q_2 < Q_{2,M'}$. In this case the monopolist believes that she will purchase a new unit in period 2 when, in fact, she will purchase a new unit in period 3. Nevertheless, she expects the monopolist to extract all the surplus in selling a new unit in period 2 so her willingness to pay for a new unit in period 1 is again $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. As before, this means surplus is shared and monopoly profit is lower than in the case $\beta = 1$. This completes the proof of all the statements in the proposition concerning monopoly.

PROOF OF PROPOSITION 7 Given the parameter restriction $v1Q_1 > c$, in equilibrium all consumers purchase a new unit in period 1. We first consider the case of perfect competition. The zero profit condition associated with perfect competition means that the new-unit price in each period is c.

Consider again consumer i. Given $v(Q_3 - Q_1) < c - z$, the consumer won't purchase a new unit in period 3 independent of whether or not the consumer purchases a new unit in period 2. At the beginning of period 2 consumers will anticipate this. If the consumer does not purchase a new unit in period 2, then her discounted utility over periods 2 and 3 equals $(1 + \delta)vQ_1$. If the consumer purchases a new unit in period 2, then her discounted utility over periods 2 and 3 equals $(1 + \delta)vQ_2 - (c - z)$. Equating the two expressions yields that the consumer is indifferent between purchasing and not purchasing a new unit in period 2 if $Q_2 = Q_2^{**}$. Given discounted utility over periods 2 and 3 rises with Q_2 if the consumer purchases a new unit in period 2 and is independent

of Q_2 if the consumer does not, we now have that the consumer purchases a new unit in period 2 if $Q_2 > Q_2^{**}$ and does not purchase a new unit in period 2 if $Q_2 < Q_2^{**}$. Also, given the new-unit price is c in each period, all surplus is received by consumers. This completes the proof of all the statements in the proposition concerning perfect competition.

Now consider the case of monopoly. Independent of whether a consumer purchases a new unit in period 2, maximum willingness to pay in period 3 is less than or equal to $v(Q_3 - Q_1) < c - z$, which means the monopolist does not sell new units in period 3 whether or not consumers purchase new units in period 2.

Given this, consider the beginning of period 2. Consumer willingness to pay for a new unit at the beginning of period 2 equals $(1+\delta)v(Q_2-Q_1)+z$, which is the price if the monopolist sells new units in period 2. Thus, discounted monopoly profit over periods 2 and 3 associated with selling new units in period 2 is positive (negative) if $Q_2 > (<)Q_2^{**}$, which in turn means the monopolist sells (does not sell) new units in period 2 if $Q_2 > (<)Q_2^{**}$. Finally, whether Q_2 is less than or greater than Q_2^{**} , in period 1 the monopolist charges willingness to pay which in both cases equals $(1 + \delta + \delta 2)vQ_1$. Note, since whenever consumers purchase the price extracts all the surplus, in both cases all surplus is received by the monopolist. This completes the proof of all the statements concerning monopoly.

PROOF OF PROPOSITION 8 Given the parameter restriction $vQ_1 > c$, in equilibrium all consumers purchase a new unit in period 1. We first consider the case of perfect competition. The zero profit condition associated with perfect competition means that the new-unit price in each period is c. Consider consumer i. Given $v(Q_3 - Q_1) < c - z$, the consumer won't purchase a new unit in period 3 independent of whether or not the consumer purchases a new unit in period 2. At the beginning of period 2 consumers will anticipate this behavior. If the consumer does not purchase a new unit in period 2, then her perceived discounted utility over periods 2 and 3 equals $(1+\beta\delta)vQ_1$. If the consumer purchases a new unit in period 2, then her perceived discounted utility over periods 2 and 3 equals $(1 + \beta \delta)vQ_2 - (c - z)$. Equating the two expressions yields that the consumer is indifferent between purchasing and not purchasing a new unit in period 2 if $Q_2 = Q_{2,P''} = Q_1 + [(c-z)/(1+\beta\delta)v] > Q_2^{**}$. Given discounted utility over periods 2 and 3 rises with Q_2 if the consumer purchases a new unit in period 2 and is independent of Q_2 if the consumer does not, we now have that the consumer purchases a new unit in period 2 if $Q_2 > Q_{2,P''}$ and does not purchase a new unit in period 2 if $Q_2 < Q_{2,P''}$. Given the new-unit price is c in each period, all surplus is received by consumers. Also, given this, $Q_{2,P''} \neq Q_2^{**}$, and the definition of Q_2^{**} , both social welfare and consumer welfare are below the values when $\beta = 1$. This completes the proof of all of the statements in the proposition concerning perfect competition. Now consider the case of monopoly. Independent of whether a consumer purchases a new unit in period 2, maximum willingness to pay in period 3 is less than or equal to $v(Q_3 - Q_1)$. Given $v(Q_3 - Q_1) < c - z$, this means the monopolist does not sell new units in period 3 whether or not consumers purchase new units in period 2. Given this, consider the beginning of period 2. Consumer willingness to pay for a new unit in period 2 equals $(1+\beta\delta)v(Q_2-Q_1)+z$ which is the price if the monopolist sells new units in period 2. Thus, discounted monopoly profit over periods 2 and 3 associated with selling new units in period 2 is positive (negative) if $Q_2 > (<)Q_{2,P/M''} > Q_2^{**}$, which in turn means the monopolist sells (does not sell) new units in period 2 if $Q_2 > (<)Q_2^{**}$.

Consider the cases $Q_2 < Q_2^{**} and Q_2 > Q_{2,P/M''}$. The period-2 price for a new unit when sales occur in period 2 is given above. In the case $Q_2 > Q_{2,P/M''}$ the naïve consumer correctly believes that she will purchase a new unit in period 2 and she also believes the monopolist will extract all the surplus. Thus, her period-1 willingness to pay for a new unit is $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. In the case $Q_2 < Q_2^{**}$, the naïve consumer believes that she will not purchase a new unit in period 2, and that if she does not purchase a new unit in period 1 then she will purchase a new unit in period 2 and the monopolist will extract all the surplus. Thus, her period-1 willingness to pay for a new unit is again $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. Note that these prices are less than in the case $\beta = 1$ which means surplus is shared and monopoly profit is lower than in the case $\beta = 1$.

Finally, consider the case $Q_2^{**} < Q_2 < Q_{2,P/M''}$. In this case the monopolist believes that she will purchase a new unit in period 2 when, in fact, she will not. Nevertheless, she expects the monopolist to extract all the surplus in selling a new unit in period 2. So her willingness to pay for a new unit in period 1 is again $(1 + \delta\beta + \delta2\beta)vQ_1$ which is the price. As before, this means surplus is shared and monopoly profit is lower than in the case $\beta = 1$. This completes the proof of all the statements in the proposition concerning monopoly.