Vertical Differentiation, Branding, and Product Confusion*

Thomas Jungbauer† Chenyang Li‡

Sherif Nasser§ Christian Schmid¶

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Abstract

The decision whether a multi-product firm offers its goods under a joint or separate brands is essential for its success. When selling vertically differentiated products, it needs to consider pricing, cannibalization, and—when branding jointly—product confusion, i.e., consumers associating characteristics with the wrong product. The extent of confusion crucially depends on the accuracy of available information. We study the branding and pricing problem of a firm selling vertically differentiated products to a mix of naive and sophisticated consumers. The analysis accounts for the spillover arising from product confusion. Our findings suggest that joint branding is optimal when the spillover is either high or low but not in between. When low, firms jointly brand to save the cost of erecting a second brand. In contrast, when spillover effects are high, the firm brands jointly since it is inherently more profitable even if building additional brands is free. In between, firms opt for separate branding despite the additional cost of building more brands. We also find that a higher fraction of sophisticated consumers does not necessarily push the firm towards joint branding. Moreover, firms are inclined to jointly brand more similar products, and, if doing so, decrease price dispersion.

Keywords: Vertical differentiation, product confusion, branding, cannibalization, information provision

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†Cornell University, jungbauer@cornell.edu;

‡Hong Kong University of Science and Technology (HKUST), chenyangli@hkust-gz.edu.cn ;

§Cornell University, szn2@cornell.edu;

¶IMC Krems University of Applied Sciences, christian.schmid@imc.ac.at;

1 Introduction

Managing consumer perceptions and expectations about product value poses a significant economic issue in a wide variety of industries, even more so in online retailing (Fan et al., 2020). In today's marketplace, consumers are exposed to a flurry of information regarding the benefits and detriments of products before making their purchasing decisions. Based on these influences—such as online reviews, commercials, word-of-mouth advertisement and past experience—consumers form an expectation about a product's quality. If this product is sold under its own *separate* brand, and the data to which consumers are exposed is sufficiently rich, there is no immediate reason to assume this expectation to be biased. When products of different quality are sold, however, under a *joint* brand, consumers may be affected by *product confusion*, that is their expectation about one product is biased towards the quality of other products (Parker, 2010). This phenomenon may arise either when the sender of information, e.g., the author of an online review, or the receiver of information, e.g., the reader of such review, associate characteristics with the wrong product. Product confusion is a special case of consumer confusion (Sertoglu and Kavak, 2017), well known in the pharmaceutical industry due to its potentially dire consequences (see e.g., Agrawal, 1999).[1]

The magnitude of this bias, i.e., how much quality "spills over" from one product to another, depends inversely on the accuracy of information consumers are exposed to. While verified online customer reviews for example tend to be fairly accurate in identifying the correct product, the accuracy of word-of-mouth, advertisements, or past experiences may vary wildly. This gives rise to the first main research question we address in this paper. How does the extent of product confusion, henceforth the *spillover*, affect the branding decision of a firm selling vertically differentiated products, that is when cannibalization matters? Naturally, consumers vary in terms of their cognitive abilities (Yoo and Sarin, 2018), attentiveness to information (Ghosh and Galbreth, 2013), and available resources. We simplify by modeling consumers to be either naive–not aware of any bias arising from joint branding—or sophisticated, i.e., consumers who know about and correct for

[1]Throughout this paper we refer to product confusion—the inability to accurately match characteristics exhibited by some product(s) of a given brand with the correct product—in contrast to brand confusion—the inability to distinguish between similar products by different brands. Brand confusion gives rise to trademark law (Lemley and McKenna, 2010), clearly not applicable under product confusion when the products under consideration are sold by the same entity. Our definition of product confusion relates to Item 15: "I feel confused when different names are given to different models of the same brand, like automobiles," in the factor analysis and consumer survey of Sertoglu and Kavak (2017) about confusion in the marketplace.

product confusion. This allows us to address our second main question. How does the optimal branding regime of a vertically differentiated firm depend on the sophistication of its consumers?

Our main results indicate that the optimal branding strategy is in general non-monotone in the spillover, that is more accurate information does not always push the firms towards joint branding or vice versa. The firm prefers to brand its products jointly if the spillover among them is small to forego the cost of erecting a second brand, or large to boost a lower quality product at the expense of a higher quality one. Sophisticated consumers correctly derive true product quality. Nevertheless—despite not having to erect a costly second brand—a higher fraction of sophisticated consumers does not necessarily push the firm towards joint branding due to to a pricing problem arising when both types of consumers are sufficiently prevalent. The interplay of the spillover and the fraction of sophisticated consumers is depicted in Figure 1. If either the spillover is low or the fraction of sophisticated consumers is high, the firm prefers to brand jointly due to cost motives. On the contrary, when the spillover is high and the the fraction of sophisticated consumers is low, the firm prefers to brand jointly as well because it benefits from obfuscating its naive consumers. In between, however, a separate branding regime maximizes the firm's profits.

Fraction of sophisticated consumers

Spillover	Low	Intermediate	High
H	Joint	Separate	Joint
I	Separate	Separate	Joint
L	Joint	Joint	Joint

Figure 1: Optimal branding regime

Our results also support the conventional wisdom that a firm is more likely to jointly brand more similar products. Moreover, the firm always reduces price dispersion when branding jointly. Then, we characterize conditions under which a firm resolves uncertainty to avoid product confusion even if doing so is costly, e.g., through facilitating free returns. In addition, we show that similarly vertically differentiated firms may choose asymmetric branding regimes in order to differentiate from each other, a potential explanation for real-world observations of competing vertically differentiated firms

with different branding strategies such as automobile manufacturers. Finally, low-end (respectively high-end) competition such as tussling with a store-brand pushes the firm towards joint branding in our model, as the need to differentiate increases.

Following (Eckel and Neary, 2010), multi-product firms make up for slightly less than half of the total number of firms and account for 90% of sales figures globally. What is more, these firms are a driving force behind the diverse and dynamic global product offerings as the vast majority of them update their product mix every five years on average. A significant fraction of these multi-product firms sell vertically differentiated products, i.e., imperfect substitutes with respect to product quality within the same category (Shapiro, 1994; Johnsen, 2018; Behrmann, 2019). Among the most critical strategic decisions such a firm faces, is whether to advertise and price its products under a joint or separate brands. Some follow a "branded house" approach and market their products under a joint brand. Examples of this strategy are witnessed in industries from personal computing devices (offering competing tiers of laptops such as Dell with its XPS and Latitude series) to fashion (i.e., Polo Ralph Lauren vs. Ralph Lauren Purple Label). Others, in contrast, run a "house of brands," that is to say they produce and sell under separate brands. Some examples exercising this branding strategy are conglomerates such as Samsung (Samsung branded vs. the high-end Dacor kitchen appliances, etc.) but also Japanese car manufacturers such as Nissan (Infiniti vs. Nissan).

What drives this difference in branding decisions? The opportunities and perils of joint branding are multifold. Some firms opt for a branded house due to positive brand spillovers, while others model themselves as a house of brands when negative brand spillovers predominate.[2] Direct positive spillovers arise from straightforward product enhancement by complementarity, exclusivity or reputation, while direct negative spillovers result from tarnishment, blurring or conspicuous consumption. Indirect spillovers, on the other hand, are transmitted via the brand as a vehicle of information. Branding can help firms reduce informational asymmetries, or hurt them by increasing their operational risk through association of different products. In this paper, we focus on the informational spillovers resulting from product confusion which obfuscate product characteristics rather than resolve uncertainty. This type of spillover arises when consumers are not certain which

[2]The presence of positive as well as negative brand spillovers is well established in the literature (see Keller and Lehmann (2006) and Gabaix (2014) among others).

information is associated with which of the products sharing a joint brand. These spillovers among jointly branded products are particularly important in markets in which the majority of purchases are not undertaken by repeat customers on a frequent basis.[3] Prime examples are markets for durable and expensive goods, such as the markets for cars, televisions, personal computing devices, investment fashion pieces such as winter coats, etc., or markets for expensive experience goods such as the cruise or theme park industry.

The trade-offs between these branding regimes are evident in executive decision-making around the world. The French luxury leather company Longchamp, for example, is well known for associating fashion with premier horseracing. Its most successful product, however, is the accessible women's foldable handbag Le Pliage. While Le Pliage's sales figures undoubtedly benefit from Longchamp's exclusive and upscale firm image, the company's premier products have been struggling now for a while to distance themselves from the successful mass product (Avery, 2017). Similarly, Korean auto maker Hyundai decided to spin off its luxury vehicle Genesis as a stand-alone brand after it had struggled to gain a foothold in the luxury car industry (Nam, 2015).

In this paper, we present a novel game-theoretic model to investigate the strategic decisions of branding and pricing vertically differentiated products when informational spillovers arise due to product confusion. We study a firm that sells two products of different qualities and chooses whether to brand them jointly, selling to a mix of naive and sophisticated consumers that value quality heterogeneously in the sense of Mussa and Rosen (1978) and Moorthy (1988). Each consumer processes a large set of data points—one for each product—and aggregates these data points into signals indicating the quality of the respective product. Under a separate branding regime, we assume these signals to be unbiased and reflect true product quality. When the firm decides to brand its products jointly, however, the story is a different one. In this case, some of the data points a consumer processes when forming a quality signal emanate from the wrong product.

When a product is sold under a separate brand, its ex ante perceived quality is independent of the other product and the products solely interfere via means of cannibalization. We focus on the branding and pricing decisions of a firm accounting for the positive spillover of its premium onto its lower-tier product as well as the negative spillover working in the other direction. Defining the ratio of quality to cost differences between the products as the relative efficiency of the premium

[3]For an analysis of the branding decision when consumers undertake repeat purchases see Cabral (2009).

product then allows us to identify optimal branding and pricing behavior as a direct function of the spillover, the tradeoff between the product lines and the cost of erecting a distinct brand. Moreover, we analyze how the mix of naive and sophisticated consumer affects the firm's decision making. Naive consumers do not account for the effect of product confusion on quality signals. Therefore, these quality signals represent a naive consumer's ex ante perceived qualities of the products, while sophisticated consumers are well aware of the spillover arising from product confusion.

The effect of informational spillover on the optimal branding strategy is in general non-monotone. Firms may want to brand jointly for either of two distinct and mutually exclusive reasons. First, when spillover effects are small, firms optimally brand jointly avoiding the cost of eecting a second brand. In this scenario, the jointly branding firm benefits from minimal product confusion. Second, if the spillover is sufficiently large, the optimal course of action is to jointly brand to boost the lower-tier product. In this case the jointly branding firm is willing to forego (some) profits of the premium product and benefits from substantial product confusion.[4] When spillover effects are intermediate, however, the firm optimally never opts for a joint branding regime.

While this non-monotonicity may seem counter-intuitive on the first glance, the mechanism behind it is fairly robust. An increase in the spillover—i.e., the lower (higher) quality product benefits (suffers) more from association with a joint brand—has two major consequences. First, the larger the spillover between the products, the higher is the margin the firm transfers from the high to the low-quality product. We call this the *margin transfer effect*. Second, as the spillover increases, so does the *demand cannibalization effect*, that is substitution of high-quality product sales by the lower quality alternative. While the margin transfer effect increases in the spillover, the demand cannibalization effect decreases. It follows that there is a critical spillover such that the marginal profit of joint branding decreases up to that value and increases thereafter.

Likewise, the firm's optimal branding strategy is in general not necessarily monotone in the fraction of sophisticated consumers. Under separate branding the quality signals for both the premium and the lower tier product are unbiased. On the contrary, when the firm brands jointly, these signals are tainted by product confusion affecting the ex ante quality perception of naive consumers. Sophisticated consumers correct for product confusion, and, as a result, are indifferent

[4]The optimality of operating a premium product under a joint brand even at a loss is in line with the results of Wernerfelt (1988), Kort et al. (2006) and Hakenes and Peitz (2008) among others.

between branding regimes for given prices. The cost of erecting a new brand suggests that an increase in the fraction of sophisticated consumers always pushes the firm towards joint branding. This intuition, however, is misleading. Unable to price discriminate between consumer types, the firm faces a dilemma under joint branding whether to cater their pricing policy to naive or sophisticated consumers. This pricing dilemma is at its worst if both fractions of sophisticated and naive consumers are substantial. Thus, firm profits when branding jointly are generally convex in the the fraction of sophisticated consumers. When the market is split equally between sophisticated and naive consumers, the firm faces a pricing inefficiency in selling to two equally important segments with different quality perceptions. This makes joint branding unattractive when the fraction of sophisticated consumers is intermediate.

Due to the enormous significance of the branding decision for firm success, there is a myriad of opinions circulating on how managers should make these decisions (as discussed by Amaldoss and Jain, 2015). According to conventional wisdom, a joint branding regime is a bad idea if the the products under scrutiny are too dissimilar. As such, the savvy choice is to avoid overstretching the brand, a stance that has been corroborated by Tauber (1988), Aaker and Keller (1990), Keller and Lehmann (2006) and Pullig et al. (2006). This claim is also supported by our finding that a very efficient premium product drives the firm towards separate branding. The more efficient the premium product, the less it pays to forego its potential profits to boost the lower-tier alternative. We identify a pair of parameter values that describe the optimal branding strategy. Whenever the relative efficiency of the premium product falls short of the lower value, the firm jointly brands and whenever it exceeds the higher value, it does not. In between, the firm maximizes its profits by the branding approach described above. Equilibrium prices are less disperse when the firm opts for joint branding. Due to product confusion, joint branding makes products appear more similar for naive consumers. In addition, equilibrium pricing further reduces differences among them.

We then investigate when a firm does has an incentive to voluntarily disclose the quality of its products in order to increase its profits. We find that if information accuracy is high to begin with, i.e., product confusion is limited, the firm wants to resolve any remaining uncertainty about product qualities if the cost of doing so is affordable. When product confusion is substantial, however, the firm is better off keeping naive consumers in the dark about its products' true quality.

Competing firms generally face an incentive to differentiate in order to increase their respective

profits (see e.g., Shaked and Sutton 1982). When two firms with similar product lines compete, they may choose branding as a means of differentiation. In equilibrium—if the spillover is sufficiently high—one firm opts for separate branding, while the other brands its products jointly. Consequently, the need for differentiation may arguably push firms with similar products towards different branding regimes in real-world markets such as the car industry. On the other hand, many firms that offer vertically differentiated product lines face low-end competition such as private labels offered by traditional and online retailers. Analyzing the optimal branding strategy in the presence of a strategic low-end competitor yields an interesting result. We find that this kind of competition pushes the multi-product line firm always towards joint branding. While a firm may opt to jointly brand to optimally solve its cannibalization problem, when competing the firm exhibits an additional benefit. As the firm's low-quality product gets attacked from below, strengthening its perceived quality becomes relatively more important than in the scenario without a competitor. Joint branding allows the firm to more distinctly differentiate itself, which in turn relaxes competitive pricing pressures on its entire product line. Thus, whenever a firm jointly brands, it will do so under competition.[5]

2 Related Literature

This paper contributes to several literature streams. First, we introduce a novel theoretical model of brand spillover that relies on product confusion. Second, it is one of only a handful of papers that analyzes the branding of vertically differentiated product lines. And third, we contribute to the literature of information provision bridging brand management and consumer uncertainty.

Brand spillover The fact that consumers confuse products of a given brand, i.e., associate certain characteristics with the wrong product is well known (Sertoglu and Kavak, 2017), particularly in the medical and pharmacological world, in which product confusion may lead to dangerous consequences Agrawal (1999). To the best of our knowledge, this paper is the first that introduces a theoretical model to analyze how product confusion affects the optimal branding regime of firms. Other reasons why brand extensions are often desirable and therefore ubiquitous are complex and multi-fold. The existing literature broadly identifies three coarse justifications for introducing a

[5]While not explicitly modeled, the same logic applies if the firm faces a high-end competitor.

new product under an existing brand. First, erecting and establishing a new brand is massively expensive (Aaker and Keller, 1990) due to a lack of economies of scope (costs, customer reach, marketing effectiveness, etc.). Second, a well-known brand may act as a vehicle of information. Third, as argued by Pepall and Richards (2002), even in the absence of signaling and economies of scope, customers are often willing to pay more for a brand due to complementarities in consumption (analagous to Becker and Murphy (1993) for advertising), because of preferences to consume brands well-known or sported by others (Amaldoss and Jain, 2015). This is particularly true for industries in which product functionality is less important (Kort et al., 2006). Moreover, consumers may be willing to pay more for a brand related to their self-image (see e.g., Fazli and Shulman, 2018). Another motivational argument for the extension of a successful brand that commands power in a given industry is to strengthen that market position by diversification, i.e., line extensions within the existing product category, or to extend it to other markets.[6]

The most closely related stream of papers that concerns spillovers focuses on the brand as a vehicle of information, serving as a means to signal quality or other characteristics of a new product and to reduce informational asymmetries or increase awareness. Wernerfelt (1988) for example argues that a brand can serve as a reputational bond to guarantee product quality if the bond is perceived to be substantial.[7] While undoubtedly important, the nature and validity of the quality signaling argument is in fact highly contested. Choi (1998) and Cabral (2000) also present theoretical evidence for brand stretching as an indicator of high quality via reputation. Hakenes and Peitz (2009), in the same vein, see branding as a potential substitute for external certification and Chen and Lai (2010) substantiate a positive reputation stretching effect in the mutual funds market. Erdem (1998), however, claims that spillover effects from joint branding are complex and not well enough understood despite the vast Marketing literature on transferability of associations. Rasmusen (2016), for example, shows that it may take market power in at least one product category to signal quality across markets with a joint brand, while Hakenes and Peitz (2008) argue that due to a multiplicity of equilibria, joint branding does not necessarily guarantee high quality. Miklos-Thal (2012) argues that associating high-margin products that can fail independently (see also DeGraba and Sullivan, 1995) may not be in the best interest of the firm. Moorthy (2012) calls

[6] This argument is similar in vein to the process of tying described in Carlton and Waldman (2002).

[7] Johnson (2013) argues that firms actively seek and value to be the bondposter in a vertical inter-firm relationship.

for more research indicating that the game-theoretical foundation of the signaling power of joint branding relies solely on questionable off-equilibrium beliefs.

Vertical line extensions. Line extensions are varieties of existing brands in their current product classes (Kirmani et al., 1999). These extensions can be horizontal, i.e., alternative versions of the original product (e.g., a Toyota Camry and its hybrid version) or vertical, that is higher- or lower-priced versions of the existing brand product. Price-based extensions can stretch the line downwards (think of Mercedes' entry sedan CLA250) or upwards (such as the Chase Sapphire credit cards). Whereas both the Economics and Marketing literature feature a comprehensive treatment of the more general topic of brand extensions—the introduction of new products under an existing brand potentially into entirely different product categories (Aaker and Keller, 1990)—the special case of line extensions has been analyzed rather sparsely.

The vast majority of papers dealing with brand extensions describe spillover effects into potentially unrelated product markets. The few major contributions that deal with vertical line extensions include the seminal model of how to brand conspicuous goods by Amaldoss and Jain (2015) analyzing social effects among two disjoint sets of customers. Followers benefit from being associated with the same brand as snobs, who suffer. They find that a monopolist prefers a joint branding regime whenever followers value conformity more than snobs value exclusivity, and that this result may reverse due to competition. Jungbauer and Schmid (2021) focus on the sequential introduction of vertically differentiated product lines and find that branding may serve as a commitment device for competing firms to soften quality competition by limiting investment in product quality. Li (2019) studies the direction of vertical line extensions as an entry deterrent in the absence of a branding decision. None of these papers, however, focuses on product confusion as a source of branding effects, and on the interplay of branding and direct cannibalization.[8]

Information provision. In Section 6, we analyze when firms wants to disclose the quality of their products even if doing so is costly. Kuksov and Lin (2010) suggest a viable but costly way to resolve uncertainty about product quality are consumer product returns, and find that differentiated product competition may actually increase information provision. In our model, a firm also prefers to avoid obfuscation, i.e., joint branding, the more differentiated its products

[8] Jungbauer and Schmid (2021) analyze the interplay of the direction of the vertical extension and competition in a general model of branding that may also interpreted as one of product confusion. There firms, however, sell in two different markets and thus are not subject to cannibalization.

are. On the other hand, a differentiated competitor, such as low- or high-end competition, nudges the firm towards joint branding creating product confusion. In Oh and Park (2019), competition may push an incumbent towards non-disclosure. Their result, however, relies on obfuscation of information for an entrant, not consumers. Finally, Guo (2020) argues that firms may withhold information to mitigate upstream exploitation by input suppliers, while in our model firms may choose against information disclosure when they benefit from naive consumers buying the lower-tier product at a high price.

Lastly, several papers share key elements of our approach. Uncertainty about product characteristics, especially as it relates to branding, draws on Bergen et al. (1996) who argue that consumers have to remember, evaluate, and process a wider variety of product features when dealing with multiple products and Villas-Boas (2004) making the case that it is in fact more costly for a firm to communicate characteristics when selling multiple goods. Finally, Zhu and Dukes (2017) argue that consumers are often constrained in the amount of attention they can devote to evaluating products. Mixing naive and sophisticated consumers is a common approach to model heterogeneity in rationality, resources, or the ability to process information shared by Ghosh and Galbreth (2013), Yoo and Sarin (2018) and Guan et al. (2020) among others.[9] Neeman et al. (2019) analyze when competing products of different firms want to share a joint brand, characterizing a tradeoff between free-riding and disciplining behavior.

3 The Model

A firm produces two vertically differentiated products, a low-quality product l and a high-quality product h, with qualities q_l and q_h, such that $0 < q_l < q_h$, with constant marginal costs c_l and c_h, respectively. We assume that $c_l < c_h$ and, for simplicity, normalize c_l to 0.[10] The quality, q_i, $i \in \{l, h\}$, is a composite measure of product characteristics that objectively affects consumer utility (e.g, performance level, durability, luxurious materials, warranties, customer service, etc.).

The firm serves a mass of consumers who are heterogeneous in their valuation of quality and buy at most one product. Below, we allow for consumers' ex ante perception of a product's quality

[9] In Iyer and Kuksov (2010), sophisticated consumers see through firms' actions when merchandising affects perceived but not true product quality. Despite the consumers' ability, firms still invest in merchandising in equilibrium.

[10] Below we show that this assumption simplifies algebra as the firm always sets total demand to $\frac{1}{2}$ when choosing prices optimally, but does not affect the qualitative results of the paper.

to deviate from actual quality due to product confusion. Thus, we use $\hat{q}_i$ to denote the (ex ante) perceived quality of product $i \in \{l, h\}$. The utility a consumer derives from buying product i at a price p_i is given by

$$u_i = \theta \hat{q}_i - p_i, \tag{1}$$

where θ, uniformly distributed on $[0, 1]$, captures an individual consumer's marginal willingness to pay for quality. Without loss of generality, we normalize consumers' utility of not purchasing to 0.

The firm chooses whether to brand its products separately or jointly. Under a separate branding (SB) strategy, the firm sells each product under its own brand and incurs a fixed cost of B^{SB} for building two brands. Under a joint branding (JB) strategy, the firm sells both products under one and the same brand and incurs a fixed cost of B^{JB} for building this one brand. Naturally, we assume that the cost of building two brands is higher than that of building only one and define $\Delta \equiv B^{SB} - B^{JB} \geq 0$ to denote the incremental cost of building a second brand.

Consumers receive quality information for a given product from a variety of sources such as online reviews, word of mouth, past personal experiences with the product and/or brand, firm communications, etc. We assume that consumers have access to a sufficiently large number of data points indicating the quality of each product from these various sources. The average of these data points is what we refer to as the product's quality signal. When products are branded separately, the quality signal converges to the product's true quality, i.e., it is unbiased. When products are branded jointly, however, the signal diverges from the true quality. In other words, joint branding introduces a quality signal bias.

There are multiple potential sources for a quality signal bias to arise such as misplaced online reviews, vague word-of-mouth endorsements that are brand but not product specific (e.g., "I had a Samsung phone before and the camera was horrible/great"), imperfect recollection of previous consumption experiences (e.g., "I rented a VW before, it might have been a Jetta or a Passat, I am not certain"). We model this information structure as a quality signal that depends on the firm's choice of branding regime $b \in \{SB, JB\}$. Suppose each consumer is exposed to a mass of quality data points. Under separate branding, all the data points are generated from the focal product being evaluated. Hence, the average of these data points converges to the the true quality such that $\mathbb{E}[Q_i \mid b = SB] = q_i$. Under joint branding, however, a proportion β of these data points is

generated from the other product sharing the same brand name. Hence, the average of these data points is biased such that $\mathbb{E}[Q_i \mid b = JB, \beta] = (1-\beta)\, q_i + \beta\, q_j$.

Let $\tilde{q}_{i,b}$ denote the quality signal for product $i \in \{l, h\}$, such that

$$\tilde{q}_{i,b} = \begin{cases} q_i & \text{if } s = SB \\ (1-\beta)\, q_i + \beta\, q_j & \text{if } s = JB \end{cases}, \qquad \forall i, j \in \{l, h\}, i \neq j. \tag{2}$$

The top prong of Equation (2) corresponds to the separate branding case, in which the quality signal of product i is unbiased, that is to say equals its true quality and is not affected by product j. The bottom prong of Equation (2), on the other hand, describes the joint branding case, when the quality signal of product i is subject to product confusion and therefore a function not only of its true quality but also of products j's. The premise is that when the products share the same brand, product confusion biases the quality signal. The extent of this bias, the spillover, is captured by the parameter $\beta \in [0, \frac{1}{2}]$.[11] We simplify the exposition by assuming the spillover to be symmetric, i.e., the probability that a data point emanates from the wrong product is the same across products. At the end of Section 4 we elaborate why this simplification does not affect the qualitative takeaways in this paper.

At one end, $\beta = 0$ captures the case when the quality signal of a product is only based on its own objective characteristics. This extreme case for example refers to a hypothetical market in which information sharing about products solely takes place via verified non-misplaced online reviews. In this scenario, joint branding has no effect and the product's perceived quality equals its true quality. On the other end, when $\beta = \frac{1}{2}$, the signal is the same for all products that share the same brand irrespective of their true qualities. This extreme case refers to a hypothetical scenario in which consumers are unable to differentiate between product lines and simply evaluate the brand. In this case, the quality signal of every product equals the average quality of the jointly branded products. An intermediate level of β reflects how much joint branding biases the signal (vs. true quality) of a given product. The term spillover signifies that sharing the same brand spills over some quality from product h to product l. As a result, the quality signal of the high(low)-quality product ends up lower (higher) than its true quality.

[11] The assumption that customers only associate products if they share a common visible brand is customary in the literature (see Cabral (2000), Miklos-Thal (2012) and Amaldoss and Jain (2015) among others).

In reality, the quality signal does not necessarily converge to a product's true quality when a firm brands its products separately. The primary focus of this paper, though, is how the difference of perceived qualities across the two branding regimes affects the optimal firm strategy. Assuming that the quality signal equals true quality under SB allows us to focus on exactly that differential effect by abstracting away from how branding could enhance perceived quality regardless of how many products share the same brand. Expanding the analysis to accommodate a baseline quality-enhancement effect therefore unnecessarily complicates the exposition without qualitatively changing the results of the paper.[12]

A fraction, α, of consumer is sophisticated (s) while the remaining fraction, $1-\alpha$, is naive (n). Sophisticated consumers are aware of the magnitude of the signal bias caused by joint branding and correct for it. Hence, their quality perceptions always converge to the true quality regardless of the branding regime. Naive consumers are unaware of the bias and their perceived quality is formed by the quality signal. Let $\hat{q}_{i,b,k}$ denote the ex ante perceived quality for product $i \in \{l, h\}$ under branding regime $b \in \{SB, JB\}$ by a consumer type $k \in \{s \equiv \text{sophisticated}\ , n \equiv \text{naive}\}$ where $i \neq j$,

$$\hat{q}_{i,b,k} = \begin{cases} q_i & \text{if } k = s \text{ (sophisticated)}, \\ \tilde{q}_{i,b} = \begin{cases} q_i & \text{if } b = SB, \\ (1-\beta)\, q_i + \beta\, q_j & \text{if } b = JB, \end{cases} & \text{and } k = n \text{ (naive)}. \end{cases} \tag{3}$$

We label consumers as sophisticated and naive for expositional simplicity and do not necessarily imply rational/irrational consumers. The types can also be interpreted as varying in their costs for correcting the quality signal bias. Suppose consumers have to incur a cost in terms of resources such as time and/or effort to verify and weed out biased data points to correct for the signal bias. One can interpret sophisticated (naive) consumers as those who incur a sufficiently low (high) cost for such correction.

The game we analyze proceeds in the following stages: at stage 1, the firm decides whether

[12] For instance, one could expand the model by allowing for a baseline quality signal of product i when sold under brand b to be $(1+\delta_b)q_i$, where $\delta_b \geq 0$. The quality signal under SB (JB) in the upper (lower) prong of Equation (3) then becomes $(1+\delta_b)q_i$ $\left((1-\beta)(1+\delta_b)\, q_i + \beta\,(1+\delta_b)\left(\frac{q_i+q_j}{2}\right)\right)$. The branding cost (BC) can then be modeled as $BC_b = B + f(\delta_b)$ to accommodate both a fixed component B and a convex, increasing function for quality enhancement, $f(\cdot)$. Essentially, the model we present in this paper normalizes δ_b to 0. Expanding the analysis for cases when $\delta_b > 0$ sacrifices a great deal of parsimony without adding additional insight.

it will follow a separate or joint branding strategy (i.e., it chooses $b \in \{SB, JB\}$), at stage 2, the firm sets prices, and at stage 3, consumers—maximizing their respective utility functions—choose whether to buy or not, and if so, which product. The firm chooses a branding regime and prices to maxmimize profits. We solve for the subgame perfect Nash equilibrium (SPNE) by backward induction. Starting with stage 3, as consumers maximize their utility, the following demand functions arise:

$$\begin{aligned} x_{h,b} &= \alpha\left(1 - \frac{p_h - p_l}{\hat{q}_{h,b,s} - \hat{q}_{l,b,s}}\right) + (1-\alpha)\left(1 - \frac{p_h - p_l}{\hat{q}_{h,b,n} - \hat{q}_{l,b,n}}\right), \text{ and} \\ x_{l,b} &= \alpha\left(\frac{p_h - p_l}{\hat{q}_{h,b,s} - \hat{q}_{l,b,s}} - \frac{p_l}{\hat{q}_{l,b,s}}\right) + (1-\alpha)\left(\frac{p_h - p_l}{\hat{q}_{h,b,n} - \hat{q}_{l,b,n}} - \frac{p_l}{\hat{q}_{l,b,n}}\right), \end{aligned} \tag{4}$$

where, $x_{i,b}(p_i, p_j)$ denotes the demand of product i under branding regime $b \in \{SB, JB\}$, at prices p_i and p_j, $i, j \in \{l, h\}, i \neq j$.

For sophisticated consumers, the perceived product qualities are not affected by the branding regime. Accordingly, if all consumers are sophisticated, i.e., $\alpha = 1$, the equilibrium prices and demand will converge under both joint branding and separate branding. The former will be more attractive, however, because of savings on fixed branding costs.

Lemma 1. *Whenever the fixed cost of joint branding is strictly below that of separate branding ($\Delta > 0$), and the firm faces only sophisticated consumers ($\alpha = 1$) , then joint branding is the SPNE branding strategy for any β.*

To build further intuition, we focus in the following section on the case where all consumers are naive, i.e., $\alpha = 0$. This will allow us to isolate the effect of joint branding spillover on the firm's branding strategy. Thereafter, we analyze the general case when there is a mix of sophisticated and naive consumers and investigate the interaction between spillover and fraction of sophisticated consumers.

4 Branding with naive consumers

In this section, we start by analyzing the separate branding subgame and deriving its equilibrium, followed by the joint branding subgame. Finally, we derive the SPNE in branding strategies.

4.1 The Separate Branding (SB) Subgame

Separate branding follows the classic analysis of a firm selling two vertically differentiated products. Perceived qualities follow the top prong of Equation (3) and the resulting demand functions are given by Equation (4). The firm sets prices to solve

$$\max_{p_h, p_l} \left[(p_h - c_h)\, x_{h,SB} + p_l\, x_{l,SB} \right].$$

The subsequent definition allows us to reduce the difference between product characteristics for the firm to a single dimension.

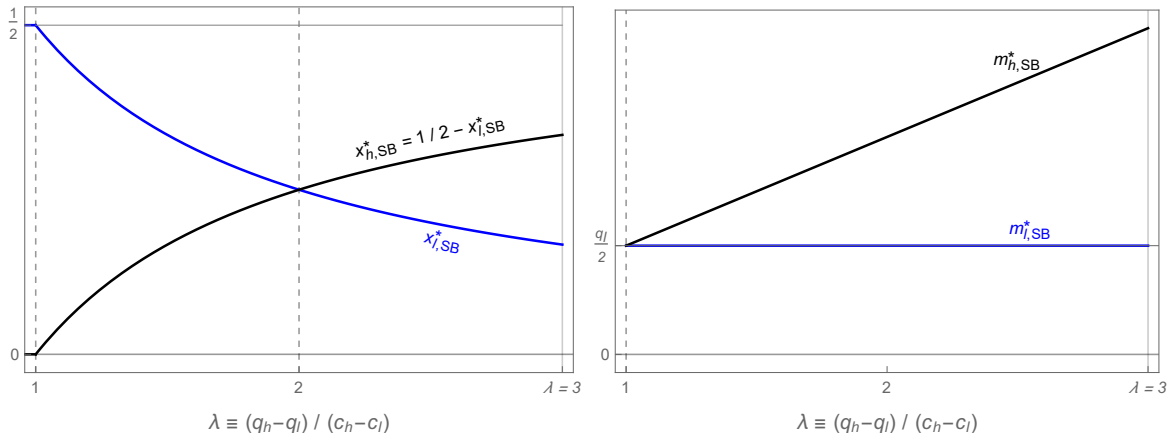
Definition 1. Let $\lambda \equiv \frac{q_h - q_l}{c_h - c_l}$ denote the *relative efficiency* of the high-quality product versus the low-quality one.

The parameter λ represents the ratio of the quality to cost differences between the two products. As it increases, the high-quality product becomes more efficient relative to the low-quality one in providing more utility to consumers relative to its additional cost of production. The definition of λ enables us to directly relate the trade-off between products to consumer demand in equilibrium. Henceforth, we will restrict attention to cases where $\lambda > 1$ to ensure that the high-quality product is efficient enough to generate positive demand under separate branding.

Lemma 2. *Under separate branding, the firm always sets the total demand for both products at* $\frac{1}{2}$ *and generates positive demand for each product. As* λ *increases, the firm allocates more (less) demand to the high(low)-quality product. The resulting subgame equilibrium prices, demand, and profits are given by Equation (5) below.*

$$\begin{aligned}
p^*_{h,SB} &= \frac{q_h + c_h}{2}, & p^*_{l,SB} &= \frac{q_l}{2}, \\
x^*_{h,SB} &= \frac{1}{2}\left(1 - \frac{1}{\lambda}\right), & x^*_{l,SB} &= \frac{1}{2} - x^*_{h,SB} \quad \textit{and} \\
\pi^*_{SB} &= \frac{1}{4}\left[q_h - c_h\left(2 - \frac{c_h}{q_h - q_l}\right)\right]
\end{aligned} \tag{5}$$

Lemma 2 presents the standard result for a firm selling vertically differentiated products. Figures 2(a) and 2(b), respectively, illustrate the relationship between λ and each product's equilibrium demand and margin. As λ increases, the high-quality product becomes more efficient relative to the low-quality one and the firm sells more (less) of the former (latter).



(a) Demand

(b) Margins ($m_i = p_i - c_i,\ i \in \{l, h\}$)

Figure 2: Subgame Equilibrium under Separate Branding

4.2 The Joint Branding (JB) Subgame

Under joint branding, the quality signals follow the bottom prong of Equation (2), which can be rewritten as

$$\tilde{q}_{h,JB} = q_h - \underbrace{\beta\,(qh - q_l)}_{\text{negative spillover}}, \qquad \text{and} \qquad \tilde{q}_{l,JB} = q_l + \underbrace{\beta\,(qh - q_l)}_{\text{positive spillover}}. \tag{6}$$

The equation above highlights how joint branding essentially causes a portion of the high-quality product to spill over to the low-quality one. The resulting demand functions follow Equation (4). The firm's sets prices to solve

$$\max_{p_h, p_l} \left[(p_h - c_h)\, x_{h,JB} + p_l\, x_{l,JB}\right].$$

Condition 1′. In the remainder of this section, we restrict attention to cases where

$$\beta \leq \overline{\beta}_n \equiv \frac{\lambda - 1}{2\lambda}.$$

This condition simply ensures that the quality spillover is not too large to ensure that the high-quality product generates positive demand under joint branding. In the appendix, we show that the main results of the paper do not qualitatively change when we relax this assumption and impose a minimum level of sales for the spillover to materialize.

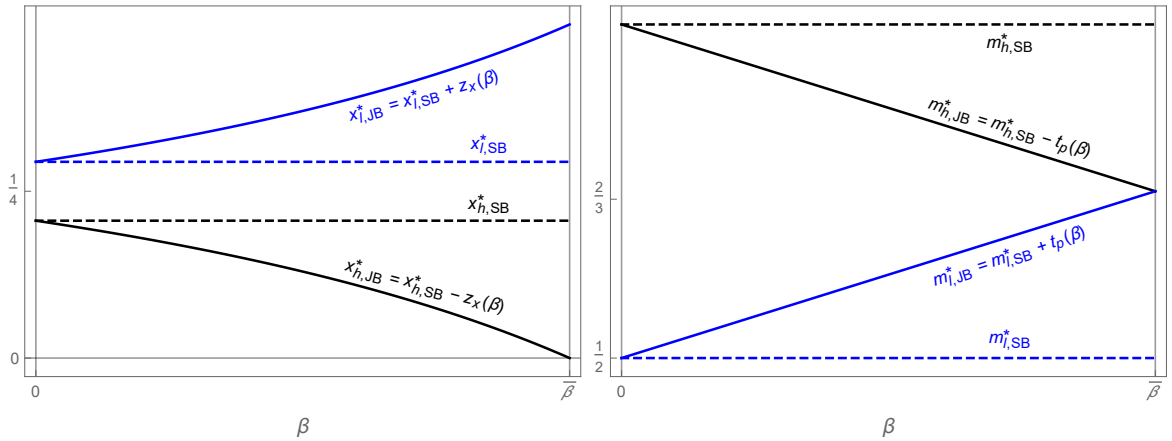
Lemma 3. *Under joint branding and only naive consumers ($\alpha = 0$), the subgame equilibrium*

demand and prices are given in Equation (7) below. The firm always sets total demand for both products at $\frac{1}{2}$, and the equilibrium demand of the low(high)-quality product is higher (lower) than its equilibrium demand level under separate branding by $z_x(\beta) > 0$, where $z_x'(\cdot) > 0$.

$$\begin{aligned} x^*_{h,JB} &= x^*_{h,SB} - z_x(\beta), & p^*_{h,JB} &= p^*_{h,SB} - t_p(\beta), \\ x^*_{l,JB} &= x^*_{l,SB} + z_x(\beta), & p^*_{l,JB} &= p^*_{l,SB} + t_p(\beta), \end{aligned} \tag{7}$$

where $z_x(\beta) \equiv \frac{1}{\lambda}\left(\frac{\beta}{1-2\beta}\right)$, *and* $t_p(\beta) \equiv \left(\frac{q_h - q_l}{2}\right)\beta$.

Figures 3(a) and 3(b) graphically illustrate the effect of the spillover, β, on equilibrium demand and margins under joint branding. When $\beta = 0$, the perceived quality of either product is the same under joint and separate branding. Hence, the equilibrium demand and margins (prices) are the same. As β increases (up to $\overline{\beta}$), the perceived quality of product h goes down, while that of product l goes up (see Equation (6)). Therefore, the firm transfers $t_p(\beta)$ from the high-quality product's margin (price) to the low-quality product's margin (price). In addition, it lets the low-quality product cannibalize the high-quality one by $z_x(\beta)$ units (relative to separate branding). Both functions $z_x(\beta)$ and $t_p(\beta)$ are increasing in β. That is to say, the higher the spillover, the higher the magnitude of both margin transfer and demand cannibalization.



(a) Demand

(b) Margins ($m_i = p_i - c_i,\ i \in \{l, h\}$)

Figure 3: Subgame Equilibrium under Joint Branding

Note. Demand plotted at $\lambda = 1.7$. Margins plotted at $q_l = 1,\ q_h = 2.7,\ c_h = 1 \Rightarrow\ \lambda = 1.7$.

We now turn our attention to how the spillover β affects the firm's profit's under joint branding.

Let $m^*_{i,JB} \equiv p^*_{i,JB} - c_i$ denote the unit margin for product $i \in \{l, h\}$. The equilibrium profit function is continuous and piece-wise differentiable in β,[13] such that

$$\frac{\partial \pi^*_{JB}}{\partial \beta} = \overbrace{t'_p\left(x^*_{l,JB} - x^*_{h,JB}\right)}^{\text{margin transfer effect}} - \overbrace{z'_x\left(m^*_{h,JB} - m^*_{l,JB}\right)}^{\text{demand cannibalization effect}} \lessgtr 0 \tag{8}$$

Corollary 1. *Under joint branding, the relationship between the subgame equilibrium profits and branding spillover, β, is as follows:*

(i) If $1 \leq \lambda \leq \sqrt{2}$, the equilibrium profits are increasing in β.

(ii) If $\lambda > \sqrt{2}$, the equilibrium profits are U-shaped (i.e., decreasing then increasing) in β.

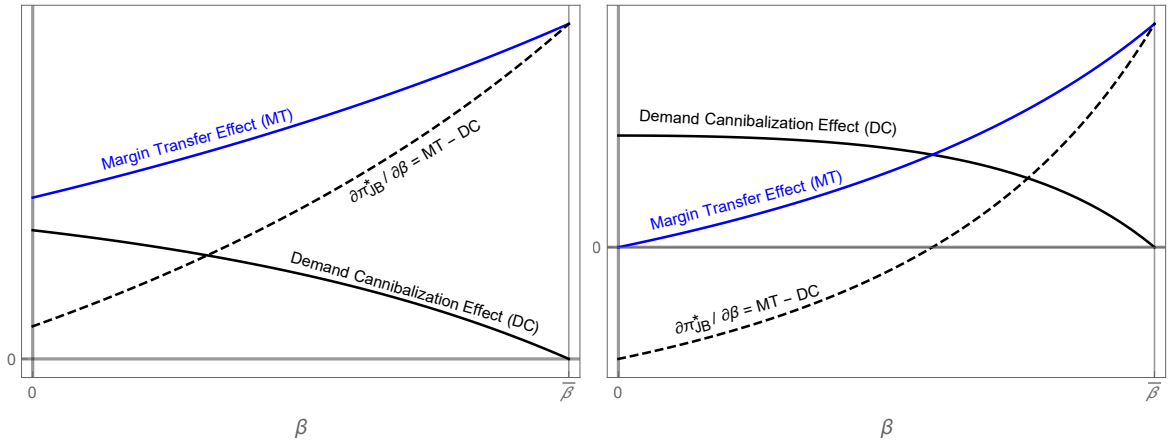
To grasp the intuition behind Corollary (1), consider Equation 8. An increase in β within the range $[0, \overline{\beta}_n)$ has two effects on profits. First, as β goes up, the margin transferred from product h to product l increases. This margin transfer effect is captured by the first term and is positive when product l's demand is higher than that of product h. Second, as β goes up, the demand cannibalized by product l at the expense of product h also increases. This demand cannibalization effect is captured by the second term and is negative when product h's margin is higher than that of product l. Therefore, profits increase (decrease) in β when the magnitude of the positive margin transfer effect is larger (smaller) than that of the negative demand cannibalization effect. Building on the results in Equation (7), the following equation highlights how λ and β influence both demand and margin differences and, in turn, both effects.

$$\begin{aligned} \text{Margin Transfer Effect} = t'_p\left(x^*_{l,JB} - x^*_{h,JB}\right) &= t'_p\Big(\overbrace{x^*_{l,SB} - x^*_{h,SB}}^{\text{decreases in } \lambda} + \overbrace{2\, z_x(\beta)}^{\text{increases in } \beta}\Big). \\ \text{Demand Cannibalization Effect} = z'_x\left(m^*_{h,JB} - m^*_{l,JB}\right) &= z'_x\Big(\underbrace{m^*_{h,SB} - m^*_{l,SB}}_{\text{increases in } \lambda} \underbrace{-\ 2\, t_p(\beta)}_{\text{decreases in } \beta}\Big). \end{aligned} \tag{9}$$

Part (i) of Corollary 1 states that if $\lambda \leq \sqrt{2}$, the positive margin transfer effect dominates the negative demand cannibalization effect. Hence, profits always increase in β. Figure 4(a) graphically illustrates the magnitude of both effects when $\lambda \leq \sqrt{2}$, and Figure 5(a) illustrates the profits. When product h is not too efficient, it captures much less demand than product l (see Figure 2(a) and Equation (9)). thus, the margin transfer effect is large and positive. In addition, the difference in equilibrium margins of the two products $(m^*_{h,JB} - m^*_{l,JB})$ is small (see Figure 2(b) and Equation

[13] The derivative exists for all $\beta \in [0, \beta_x) \cup (\beta_x, 1]$.

(9)). Thus, the negative demand cannibalization effect is small.



(a) $\lambda < \sqrt{2}$ (b) $\lambda > \sqrt{2}$

Figure 4: Spillover Effect on Joint Branding Profits

Part (ii) of Corollary 1 states that if $\lambda > \sqrt{2}$, the margin transfer effect is lower (higher) than the demand cannibalization effect when β is small (large). Hence, profits first decrease then increase in β. Figure 4(b) graphically illustrates the magnitude of both effects when $\lambda > \sqrt{2}$, and and Figure 5(b) illustrates the profits. When product h is efficient enough, $\left(x^*_{l,SB} - x^*_{h,SB}\right)$ is small and $\left(m^*_{h,SB} - m^*_{l,SB}\right)$ is large (see Figures 2). Thus, the margin transfer effect is small and the demand cannibalization effect is large at $\beta = 0$.[14] This means profits decrease in β at $\beta = 0$. As β goes up, the margin transfer effect increases and the demand cannibalization effect decreases (see Equation (9)), until the former dominates the latter and profits start to increase in β.

4.3 The Equilibrium (SPNE) Branding Strategy

What about the optimal branding strategy? To build the necessary intuition, we start by examining the SPNE when the fixed cost of joint branding and separate branding are equal, i.e., $\Delta = 0$. We then follow up with the case when separate branding entails a higher fixed cost than joint branding, i.e., $\Delta > 0$, and examine how the magnitude of that difference affects the results. In the subsequent discussion, readers might find it useful to refer to Figures 5 and 6. The former illustrates the profits under both branding strategies against the spillover β at different ranges

[14]Note that $z_x(0) = t_p(0) = 0$ and thus, at $\beta = 0$, $\left(x^*_{l,JB} - x^*_{h,JB}\right)$ converges to $\left(x^*_{l,SB} - x^*_{h,SB}\right)$, and $\left(m^*_{h,JB} - m^*_{l,JB}\right)$ converges to $\left(m^*_{h,SB} - m^*_{l,SB}\right)$.

of the high-quality's relative efficiency λ, while the latter characterizes the regions under which different branding strategies are optimal.

Proposition 1. *Suppose that the fixed cost of joint branding and separate branding are equal, i.e.,* $\Delta = 0$. *The SPNE in branding strategies will depend on* λ *and* β *as follows:*

(i) When the efficiency of the high-quality product is low (i.e., $\lambda \in \Lambda_l$*), the firm chooses joint branding regardless of the spillover, i.e., for all* $\beta \in [0, 1]$.

(ii) When the efficiency of the high-quality product is intermediate (i.e., $\lambda \in \Lambda_m$*), the firm chooses separate branding if the spillover is low (i.e.,* $\beta < \beta_2|_{\Delta=0}$*), and joint branding otherwise.*

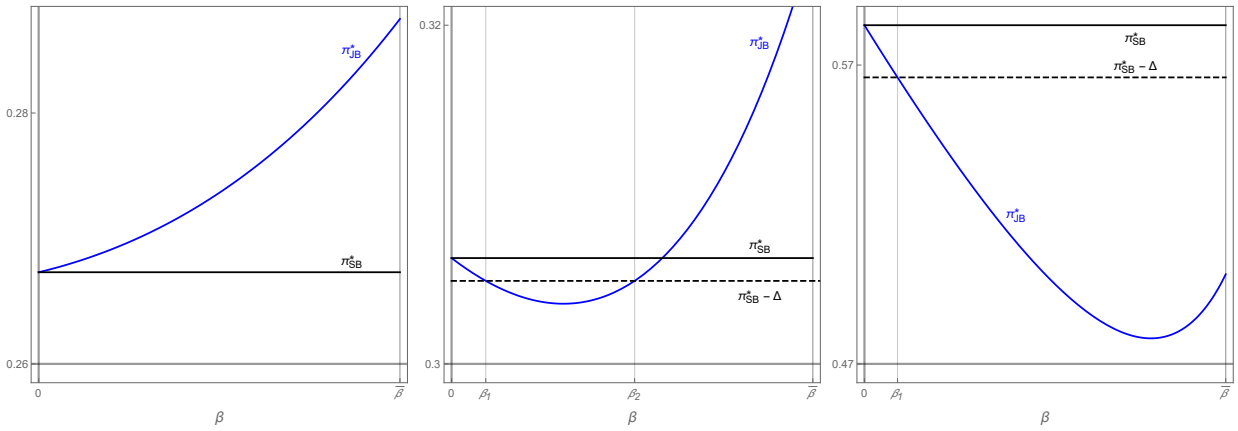
(iii) When the efficiency of the high-quality product is high (i.e., $\lambda \in \Lambda_h$*), the firm chooses separate branding regardless of the spillover, i.e., for all* $\beta \in [0, 1]$.

Λ_l, Λ_m, *and* Λ_h *are adjacent, mutually exclusive, and fully exhaustive intervals on* $[\lambda_x, \infty]$. *The spillover* $\beta_2|_{\Delta=0} \in (0, \frac{1}{2})$. *Full expressions are given in the Appendix.*

Note that all equilibrium outcomes (including profits) under joint branding converge to those under separate branding at $\beta = 0$, i.e., separate branding and $\beta = 0$ are substitutes so to say. When the efficiency of product h is low, joint branding profits always increase in β because the margin transfer benefit always outweighs the demand cannibalization loss (see the discussion following Corollary 1). Thus, the profits under joint branding are always higher than those under separate branding regardless of the spillover, β (see Figure 5(a)). When the efficiency of product h is low relative to product l, the firm finds it more profitable to jointly brand independent of the spillover to boost the perceived quality, margin and demand of product l at the expense of product h even if joint branding does not save any fixed branding costs.

When the efficiency of product h relative to l is intermediate, joint branding profits first decrease then increase in β because the margin transfer benefit outweighs the demand cannibalization loss only when β is high enough (see discussion right after Corollary 1). As such the profits under joint branding are higher than those under separate branding only when β is sufficiently high as depicted by Figure 5(b). When the efficiency of product h is intermediate relative to product l, the firm finds it optimal to utilize joint branding to boost the perceived quality, margin and demand of product l at the expense of product h only if the spillover is high enough.

Finally, when the efficiency of product h relative to l is high, joint branding profits also decrease



(a) Low λ (b) Intermediate λ (c) High λ

Figure 5: Equilibrium Profits under Joint and Separate Branding (Only Naive Consumers)

then increase in β (see discussion right after Corollary 1). However, profits under joint branding never exceed those under separate branding even when $\beta = 1$ as depicted by Figure 5(c). When the efficiency of product h is high relative to product l, the firm never finds it optimal to sacrifice its perceived quality, margin, or demand to boost those of product l and, thus, brands its product lines separately.

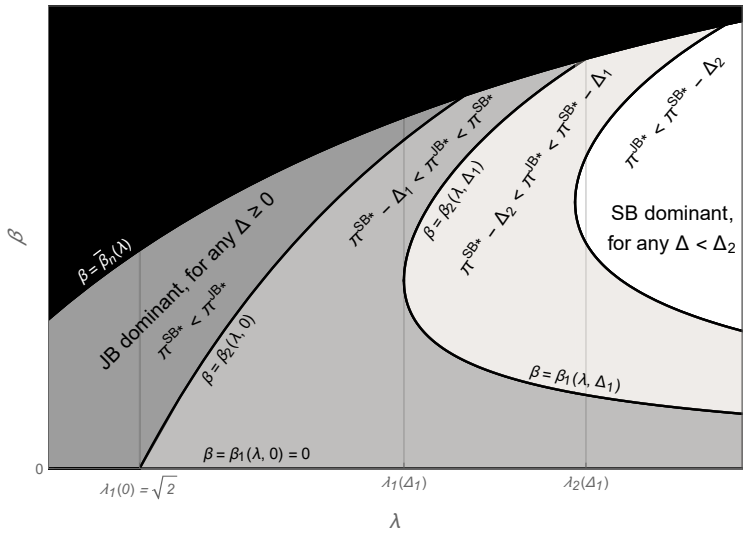


Figure 6: Optimal Branding Strategies

Next, we examine the case when the fixed branding costs are lower under joint branding than under separate branding (i.e., $\Delta > 0$). Figure 6 shows the equilibrium regions in the $\lambda \times \beta$ space. In the area with the darkest gray shading, joint branding dominates separate branding at $\Delta = 0$.

If Δ increases from 0 to $\Delta_1 > 0$, the joint branding dominance area expands to include the area with the medium shading. If Δ increases further to $\Delta_2 > \Delta_1$, the joint branding dominance area expands to include the area with the light shading, and so on.

Proposition 2. *Suppose that the fixed cost of joint branding is strictly below that of separate branding (i.e., $\Delta > 0$), and that the efficiency of the high-quality product is intermediate (i.e., $\lambda_1 < \lambda < \lambda_2$). Then, the SPNE in branding strategies will depend on the spillover, β, as follows:*

(i) The firm chooses joint branding when β is either sufficiently low or sufficiently high, i.e, $\beta \in (0, \beta_1)$ or $\beta \in (\beta_2, 1]$, respectively.

(ii) The firm chooses separate branding when β is intermediate, i.e, $\beta \in [\beta_1, \beta_2]$,

where $\beta_1(\lambda, \Delta)$, and $\beta_2(\lambda, \Delta)$ are the 2 roots that solve $\pi^{JB*}(\lambda, \beta) - \left(\pi^{SB*}(\lambda) - \Delta\right) = 0$ and are given in Equation 14. The functions $\lambda_1(\Delta)$, and $\lambda_2(\Delta)$ are defined by $\beta_1(\lambda_1(\Delta), \Delta) = \beta_2(\lambda_1(\Delta), \Delta)$ and $\beta_2(\lambda_2(\Delta), \Delta) = \overline{\beta}_n$, respectively, and are given in Equation 15.

Proposition 2 is graphically illustrated in Figures 5(b) and 6. The firm chooses joint branding over separate branding if the spillover is either sufficiently low or sufficiently high. On one hand, when β is low the firm prefers joint over separate branding in order to save on branding costs even though the latter is inherently more profitable than the former. On the other hand, when β is high the firm prefers joint over separate branding because the former is inherently more profitable than the latter.

Condition 2. Henceforth, we restrict attention to cases where the relative efficiency of product h is intermediate, such that $\lambda \in [\lambda_1, \lambda_2]$, where λ_1 and λ_2 are defined in Proposition 2. This condition ensures that the firm will choose joint branding if β is either high or low. We impose this condition to simply avoid presenting too many cases that yield the same insights.

Throughout this paper we simplify by assuming that the spillover is symmetric, i.e., the probability that a data point for the high product's quality signal emanates from the low product equals the probability that a data point for the low product's quality signal emanates from the high product. In reality, this is unlikely to hold true as information about one product may be more prevalent than information about the other. As a result, consider the following alternative specification. Let β_H denote the spillover from the low to the high product, that is the probability that a data point that contributes to the high product's quality signal emanates from the low product, and define

β_L accordingly. Assume that β_H and β_L are positively correlated. That is to say, the higher the accuracy of information in the industry, the lower both spillovers. Then, it is straightforward to show that there exist parameters $\underline{\delta_\beta}$ and $\overline{\delta_\beta}$ with $\underline{\delta_\beta} < \overline{\delta_\beta}$ such that the firm never jointly brands if

$$\beta_L - \beta_H < \underline{\delta_\beta},$$

but always does if

$$\beta_L - \beta_H > \overline{\delta_\beta}.$$

The intuition provided in this paper reflects the intermediate case of $(\beta_L - \beta_H) \in [\underline{\delta_\beta}, \overline{\delta_\beta}]$.

5 Branding with Both Types of Consumers

Now, we turn attention to the case in which the firm faces a fraction α of sophisticated consumers, and a fraction $(1-\alpha)$ of naive consumers, $\alpha \in (0,1)$. The separate branding subgame is naturally identical to that presented in Subsection 4.1.

5.1 The Joint Branding (JB) Subgame with Mixed Consumers

The firm's sets prices to solve

$$\max_{p_h, p_l} \left[(p_h - c_h)\, x_{h,JB} + p_l\, x_{l,JB} \right].$$

Condition 1. Henceforth, we restrict attention to cases where

$$\beta \leq \overline{\beta}_{mix} \equiv \frac{\lambda + \alpha(2\lambda - 1) - \sqrt{\alpha^2 + \lambda^2(1-2\alpha)^2 + 2\alpha\lambda(3-2\alpha)}}{8\alpha\lambda}, \text{ such that } \lim_{\alpha \to 0} \bar{\beta}_{mix} = \bar{\beta}_n \quad .$$

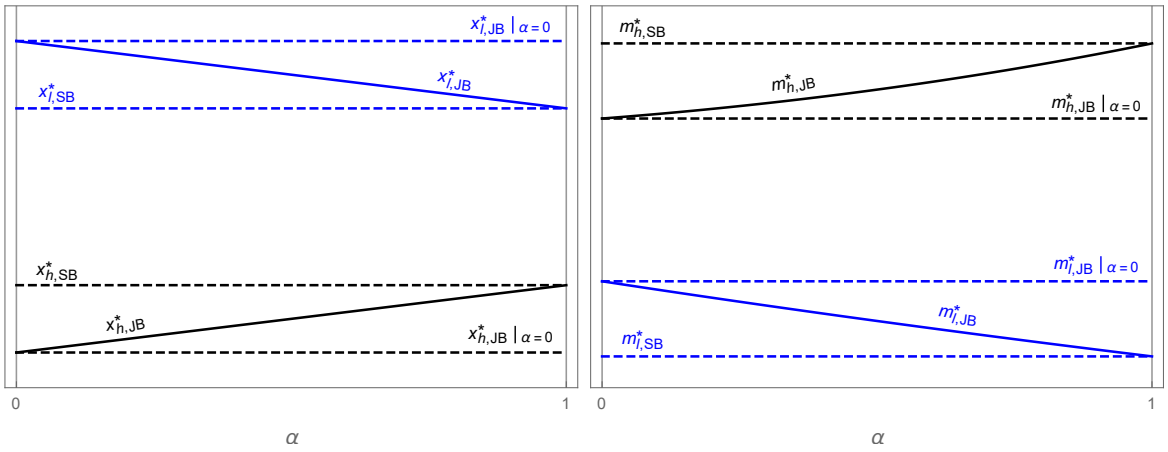
The condition above is a generalization of Condition $1'$ from the previous section. It simply ensures that the quality spillover is not too large to ensure that the high-quality product generates positive demand under joint branding in both segments.

Lemma 4. *Under joint branding with mixed consumer types ($0 < \alpha < 1$), the subgame equilibrium demand and prices are given by Equation (10). The firm always sets total demand for both products at $\frac{1}{2}$, and the equilibrium demand of the low(high)-quality product is higher (lower) than its*

equilibrium demand level under separate branding by $(1-\alpha)\, z_x(\beta) > 0$, *where* $z_x'(\,\cdot\,) > 0$.

$$\begin{aligned} x^*_{h,JB} &= x^*_{h,SB} - (1-\alpha)\, z_x(\beta), \qquad & p^*_{h,JB} &= p^*_{h,SB} - t_h(\beta), \\ x^*_{l,JB} &= x^*_{l,SB} + (1-\alpha)\, z_x(\beta), \qquad & p^*_{l,JB} &= p^*_{l,SB} + t_l(\beta), \end{aligned} \tag{10}$$

where $z_x(\beta) \equiv \dfrac{1}{\lambda}\left(\dfrac{\beta}{1-2\beta}\right)$, $t_l(\alpha) \equiv \dfrac{(1-\alpha)\beta q_l\,(q_h - q_l)}{2\,(\alpha\beta q_h + (1-\alpha\beta) q_l)}$, *and* $t_h(\alpha) \equiv \dfrac{2\alpha\beta q_h + q_l}{(1-2\alpha\beta) q_l} t_l(\alpha)$.



(a) Demand

(b) Margins ($m_i = p_i - c_i,\ i \in \{l, h\}$)

Figure 7: Subgame Equilibrium under Joint Branding with Mixed Consumers

Figures 7(a) and 7(b) graphically illustrate the effect of the fraction of sophisticated consumers, α, on equilibrium demand and margins under joint branding. When α goes to zero, consumers are naive and the equilibrium converges to that discussed in Subsection 4.2. When α goes to one, consumers are sophisticated and the ex ante perceived quality of either product is the same under joint and separate branding. Hence, the equilibrium demand and margins (prices) are the same. When α is between zero and one, the firm faces two segments with different perceptions of quality. The total equilibrium demand shifts from the low-quality product to the high-quality one as α increases. Note that the firm here cannot set the optimal price for either segment, which introduces inefficiency. The magnitude of the pricing inefficiency depends on both β and α. As β increases, the difference in perceived qualities across the two segments does as well, and thus, the pricing inefficiency becomes a more significant problem. A high or low α means that one segment is much more important than the other, which mitigates the pricing inefficiency. When α is closer to the middle rather than close to either extreme of zero or one, both segments are important and

the inefficiency is severe. In fact, there is an intermediate α^* at which the pricing inefficiency is maximized. It increases in α below α^*, and decreases above (see Figures 8(b) and 8(c)).

5.2 The Equilibrium (SPNE) Branding Strategy

What is the optimal branding strategy when the firm faces both types of consumers? Recall that joint branding dominates separate branding if $\pi^*_{JB} - \pi^*_{SB} > -\Delta$. To focus attention on cases with interesting dynamics, we will place an upper bound on the difference in fixed branding costs, Δ. Note that if Δ increases beyond this upper bound, regions where joint branding dominates simply expand.

Condition 3. Henceforth, we restrict attention to cases when $\Delta \in (0, \overline{\Delta})$, where

$$\overline{\Delta} \equiv \frac{1}{32q_l}\left(\frac{8c_h^2 q_l}{q_h - q_l} + \frac{q_h q_l^2 - 13q_l^3 - (q_h + 5q_l)\sqrt{16c_h^2 q_h q_l + 2q_l^2\left(24c_h^2 - q_h^2\right) + q_h^4 + q_l^4}}{q_h + 3q_l} + 2q_h q_l + q_h^2\right).$$

In the subsequent discussion, readers might find it useful to refer to Figures 8(a), 8(b), and 8(c), which illustrate the profit differences across both branding strategies against the fraction of sophisticated consumer, α, at different ranges of spillover, β.

Proposition 3. *Suppose that Conditions 1 through 3 hold and the firm faces a mix of sophisticated and naive consumers. The SPNE in branding strategies will depend on the spillover, β, and the fraction of sophisticated consumers, α as follows:*

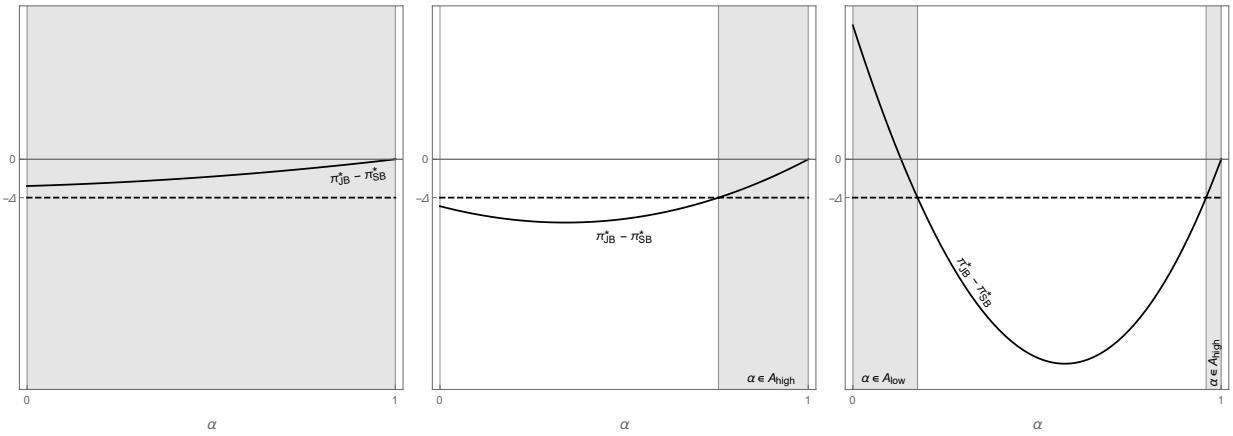
(i) When the spillover is low, $\beta \in [0, \beta_1)$, the firm always chooses joint branding regardless of the fraction of sophisticated consumers, i.e., for all $\alpha \in [0, 1]$.

(ii) When the spillover is intermediate, $\beta \in [\beta_1, \beta_2]$, the firm chooses joint branding if the fraction of sophisticated consumers is high, i.e., if $\alpha \in A_h$, and separate branding otherwise.

(iii) When the spillover is high, $\beta \in (\beta_2, \overline{\beta}_{mix}]$, the firm chooses joint branding when the fraction of sophisticated consumers is either high or low, i.e., if $\alpha \in A_l$ or $\alpha \in A_h$, and separate branding when α is intermediate, i.e., if $\alpha \in A_m$.

The mutually exclusive intervals A_l, A_m, and A_h are defined in the Appendix. The critical values β_1 and β_2 are defined in Proposition 2 and given in Equation 14.

Note that all equilibrium outcomes (including profits) under joint branding converge to those



(a) Low β (b) Intermediate β (c) High β

Figure 8: Equilibrium Profit Differences under Joint and Separate Branding (Mixed Consumers)

under separate branding at $\alpha = 1$. When the spillover is low, joint dominates separate branding as the SPNE outcome if all consumers are naive (see Proposition 2). Because β is low, the pricing inefficiency problem of serving mixed consumers is not problematic and the firm finds it more profitable to jointly brand for any α (see Figure 8(a)).

When the spillover is intermediate, separate dominates joint branding as the SPNE outcome if all consumers are naive (see Proposition 2). As α increases, joint branding becomes less profitable because the price inefficiency problem gets worse. When α is sufficiently high, however, the pricing inefficiency problem is mitigated and joint branding becomes dominant (see Figure 8(b)).

When the spillover is high, joint dominates separate branding as the SPNE outcome if all consumers are naive (see Proposition 2). In fact, joint branding is very profitable as long as most consumers are naive (α is low). As α increases, joint branding becomes less profitable because the price inefficiency problem gets worse. When α is sufficiently high, however, the pricing inefficiency problem is mitigated and joint branding becomes dominant (see Figure 8(c)) again.

6 Information Provision

Firms can choose to utilize various ways of providing accurate information to consumers to help them correct their product quality perceptions. For instance, a firm can offer free trials or free returns on its products allowing consumers to learn their true qualities (Kuksov and Lin, 2010).

Firms can also direct consumers to more accurate/unbiased information sources or support/provide these resources themselves (e.g., verified online reviews that are product rather than brand specific). In this section, we investigate the firm's incentives to provide accurate information. For simplicity, we assume that the firm can provide full and accurate quality information to all consumers at no cost, thus turning all consumers into sophisticated ones. This assumption serves to simplify the exposition and avoid case distinctions. Naturally, a firm that increases its profit by disclosing its true product qualities, however, is willing to pay for doing so as long as the incurred cost is reasonable.

Proposition 4. *Suppose that the firm, at no cost, can choose to provide full information, thereby ensuring that all consumers are sophisticated, such that joint branding is always optimal. The optimal information provision decision will depend on the spillover, β, and the proportion of sophisticated consumer, α as follows:*

(i) When the spillover is not sufficiently high, i.e., $\beta \leq \beta_2|_{\Delta=0}$, the firm always provides full information.

(ii) When the spillover is sufficiently high, i.e., $\beta > \beta_2|_{\Delta=0}$, the firm provides no information if α is low ($\alpha < \hat{\alpha}_1$). Otherwise, the firm provides full information, where $\beta_2|_{\Delta=0}$ and $\hat{\alpha}_1$ are defined in Proposition 1 and the Appendix.

If the firm provides full information, ensuring that all consumers are sophisticated, then joint branding will always dominate separate branding because of fixed cost savings (see Lemma 1). As Figures 8(a) and 8(b) illustrate, the firm always benefits from providing full information when the spillover is not large enough as the profits are highest at $\alpha = 1$.

When the spillover is sufficiently high, however, joint branding profits are inherently higher than separate branding when serving naive consumers (see Proposition 1). As Figure 8(c) illustrates, the profits when most consumers are naive (α is sufficiently small) are always higher than those when all consumers are sophisticated ($\alpha = 1$). In other words, the firm prefers to obfuscate and serve mostly naive consumers. If there is enough sophisticated consumers, however, then the firm prefers everyone to be sophisticated. Recall that the advantage of joint branding when everyone is sophisticated boils down to cost savings. This result once more underlines the two different merits of joint branding under product confusion.

7 Branding and competition

In this section, we investigate how the presence of two different types of competition affects the optimal branding decision. We start by considering symmetric competition where two similarly vertically differentiated firms compete. Thereafter, we analyze the branding decision of a focal firm with a vertically differentiated product lines when it faces a low-end competitor such as a store brand (see e.g., Nasser et al. (2013)). For tractability purposes, we will restrict attention in this section to the case where all consumers are naive.[15] The basic setup described in Section 3 in terms of joint branding spillover, marginal cost structure and consumer utility applies.

7.1 Symmetric Duopoly

For simplicity, suppose that firms cannot sufficiently differentiate themselves in terms of product quality in the sense of Shaked and Sutton (1982) and consider the case where there are two firms with similar product lines. We encounter these situation in markets in which a firm's offering has grown historically, or in those in which quality choice is discrete rather than continuous due to technological or regulatory restrictions. The intuition naturally extends to cases in which their products are sufficiently similar. They compete in a two stage game where they first simultaneously choose branding regimes, and prices thereafter.

Proposition 5. *Suppose two firms with similar product lines compete for naive consumers. If the spillover is not too low ($\beta > \tilde{\beta}$) relative to the difference in branding costs Δ, the SPNE in branding strategies is asymmetric, i.e., one firm chooses separate and the other chooses joint branding. The critical $\tilde{\beta}$ is defined in the appendix.*

When firms with similar product lines adopt the same branding strategies, they do not achieve positive profits because they compete in a Bertrand fashion and set prices equal to marginal costs. Adopting opposite branding strategies helps them differentiate their products and achieve positive profits. This arises because the firm that adopts joint branding ends up with a lower (higher) perceived quality for its high (low) quality product relative to its rival's products. Naturally, for a given branding cost differential Δ, the spillover cannot be too small ($\beta > \tilde{\beta}$) in order to generate enough differentiation to allow the firms to make up for branding costs. If β is too small, the market

[15] Numerical solutions with mixed consumers show that the results are qualitatively robust.

will not support two firms with similar product lines. This result suggests that firms offering similar products often choose diverging branding regimes in the same industry to differentiate form each other. Next, we analyze the case of low-end competition.

7.2 Low-End Competition

Now, suppose that our focal firm is facing a low-end competitor that sells a product with quality $q_e < q_l$, which it produces at zero marginal cost. As such, the focal firm has an advantage over its low-end competitor in that it can produce a better product (l) at the same marginal cost (0). The game proceeds as follows: at stage 1, the focal firm decides whether it will follow a separate or joint branding strategy (i.e., it chooses $b \in \{SB, JB\}$). At stage 2, both firms set optimal price(s) for their respective product(s). Finally, at stage 3, each consumer chooses the purchase option that maximizes her utility.

With a low-end competitor setting its price strategically, product l now faces price competition from below. Regardless of the branding strategy, standard competitive effects naturally arise as the focal firm lowers the price of product l to compete with the low-end product. In turn, it also lowers the price of product h to manage cannibalization and consumer self-selection. Our main interest, however, is to investigate whether these competitive effects can cause the firm to choose a different branding strategy when competing versus when acting as a monopolist. Therefore, the primary focus will be on comparing differences in equilibrium outcomes under separate and joint branding with competition versus these same differences without competition.

Condition 4. We restrict attention to cases where the quality of product l is sufficiently different from the competitor's low-end product, such that $0 < q_e < \overline{q}_e < q_l$. This condition arises from the following result:

Lemma 5. *Suppose the focal firm adopts a separate branding regime in the presence of a low-end competitor. Then, there exists a unique value, $\overline{q}_e \in [0, q_l]$, such that the focal firm sells both products if $q_e < \overline{q}_e$, while it sells only product h and drops product l otherwise.*

To grasp the intuition behind the result above, consider the extreme case as $q_e \to q_l$. Both products are identical and their prices will go to marginal cost and both yield 0 profits. This, in turn, exerts high pressure on the high product's price. The focal firm is better off dropping the

low product, which allows the competitor to price above marginal cost resulting in higher profits from the high product. It's important to note that when $q_e > \overline{q}_e$, joint branding always dominates separate branding, which is in line with the main result of this section that competition expands the joint branding equilibrium regions.We will assume, for the reminder of the discussion, that Conditions 1 through 4 hold.

When the focal firm chooses joint over separate branding, the spillover effect enhances the perceived quality of product l at the expense of product h. Thus, the margin and demand of product h go down, while at the same time those of product l go up. The subsequent definition will prove useful in discussing these margin/demand losses/gains.

Definition 2. Let $ML_h \equiv m_{h,SB} - m_{h,JB} > 0$ and $MG_l \equiv m_{l,JB} - m_{l,SB} > 0$, respectively denote the loss in product h's margin and the gain in product l's when the firm switches from a separate to joint branding strategy. Similarly let $DL_h \equiv x_{h,SB} - x_{h,JB} > 0$ and $DG_l \equiv x_{l,JB} - x_{l,SB} > 0$ denote the respective demand loss and gain.

We can now rewrite the results of Lemma 3 as

$$\begin{aligned} ML_h^{mon} = MG_l^{mon} = t_p(\beta) \quad &\Rightarrow \quad MG_l^{mon} - ML_h^{mon} = 0, \\ DL_h^{mon} = DG_l^{mon} = z_x(\beta) \quad &\Rightarrow \quad DG_l^{mon} - DL_h^{mon} = 0, \end{aligned} \tag{11}$$

with $t_p(\beta)$ and $z_x(\beta)$ given in Equation (7). When the firm faces no competition and switches from separate to joint branding, its margin loss on product h is equal to its margin gain on product l. Similarly, its respective demand gain and loss are equal. In other words, it reallocates demand from h to l, but it's total demand remains unchanged.

Lemma 6. *Suppose the focal firm competes with a low-end competitor. The firm's margin/demand losses/gains when it switches from separate to joint branding are given by*

$$\begin{aligned} &ML_h^{comp} = t_p(\beta) - t_p^{comp}(\beta), \quad \textit{and} \quad MG_l^{comp} = t_p(\beta) + t_p^{comp}(\beta) \\ &\qquad\qquad\qquad\qquad \Rightarrow \qquad MG_l^{comp} - ML_h^{comp} = 2t_p^{comp}(\beta) > 0. \\ &DL_h^{comp} = z_x(\beta), \qquad\qquad \textit{and} \quad DG_l^{comp} = z_x(\beta) - z_x^{comp}(\beta) \\ &\qquad\qquad\qquad\qquad \Rightarrow \qquad DG_l^{comp} - DL_h^{comp} = -z_x^{comp}(\beta) < 0. \end{aligned} \tag{12}$$

$z_x^{comp}(\beta)$ and $t_p^{comp}(\beta)$ are given in the Appendix.

Comparing Equations (11) and (12) yields useful insights that highlight how joint branding affects a competing firm differently from a monopolist. When a competing firm (as opposed to a monopolist) switches from separate to joint branding, it takes a larger gain on product l's margin and a smaller loss on product h's. The enhanced perceived quality of product l differentiates it more from the low-end competitor, which relaxes the effect of price competition. This, in turn, relaxes cannibalization pressure on product h and results in a lower loss. Interestingly, the demand loss for product h is higher than the gain for l. In other words, a competing firm loses total demand by switching to joint branding. A monopolist is competing with the outside option from below and has no reason to lose any demand when switching to joint branding. A competing firm, however, gains more by using the enhanced perceived quality of product l to disproportionately boost its margin rather than compete for low-type consumers with its competitor.

Proposition 6. *The profit difference is always strictly higher for a competing firm versus a monopolist, i.e.,* $\left(\pi^{JB*}_{comp} - \pi^{SB*}_{comp}\right) > \left(\pi^{JB*}_{mon} - \pi^{SB*}_{mon}\right)$. *Therefore, whenever a vertically differentiated monopolist chooses joint branding, so does the same firm in the presence of low-end competition. The reverse, however, is not true.*

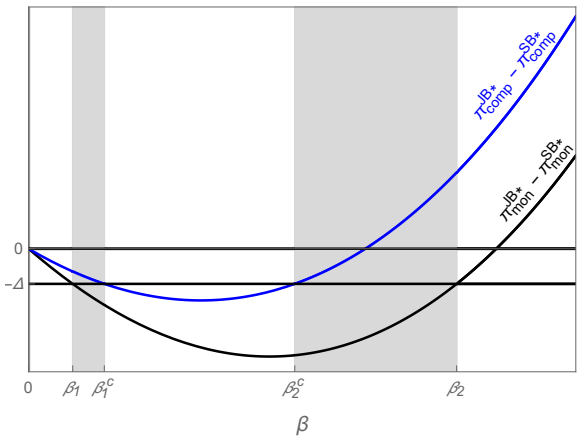


Figure 9: Branding Decision under competition versus Monopoly

The firm chooses joint branding if the difference in profits surpasses Δ, such that

$$\pi^{JB} - \pi^{SB} > -\Delta.$$

Figure 9 illustrates how that difference in profit is always higher for a competing firm versus a monopolist. Joint branding has a more positive impact on a competing firm compared to a monopolist

because it allows it to better differentiate its low-quality product against the competitor's, which in turn relaxes price competition. In the shaded regions in the figure, a competing firm chooses joint branding while a monopolist chooses separate branding.

Corollary 2. *As the quality of the competing low-end product increases, joint branding becomes more profitable relative to separate branding, i.e.,* $\pi_{comp}^{JB*} - \pi_{comp}^{SB*}$ *increases.*

As the quality of the low-end product increases, it becomes less differentiated from product l and exerts more pricing pressure, which makes joint branding more effective as in better differentiating the low product from competition and relaxing pricing pressure all across the product line. It is worth noting that the same insights arise if we analyze the focal firm competing with a high-end competitor who has a product competing with h from above. In which case, joint branding lowers the perceived quality of the focal firm's high product, thereby differentiating it further from competition.

8 Discussion

Firms that sell different products, so-called multi-product firms, are ubiquitous in today's marketplace. One of the most crucial strategic decisions such a firm faces, is whether to adopt a joint branding regime, that is to sell its products under a single brand name, or to market them as separate brands. The former strategy is often referred to as a"branded house," whereas the latter was fittingly coined a "house of brands." Both approaches are popular and observed within as well as across industries. The existing literature provides a multitude of compelling reasons for either approach. Joint branding can leverage the operation, perception and sales of a new product while such a new product line can dilute, tarnish or subject an existing brand to risk by association.

In this paper, we focus on a problem that has largely been neglected in the literature. Consumer uncertainty about product valuation combined with a joint branding regime leads to product confusion, the wrongful association of characteristics with products of the same brand. When selling vertically differentiated product lines under a joint brand, product confusion biases expected product quality towards the quality of other product lines under the same brand. Consumers differ in their ability to collect and process information. We simplify, assuming sophisticated consumers correct for the biases arising from product confusion while naive consumers do not. We investigate

how the firm's optimal branding and pricing decision varies with the extent of product confusion, i.e., the spillover, depending on the accuracy of available information.

Our analysis establishes two distinct and mutually exclusive motivations for joint branding under product confusion. If the spillover is small, i.e., the premium product only marginally boosts the perception of the lower-tier alternative and is itself not significantly impacted by the presence of this product, firms want to brand jointly to forego the cost of erecting a distinct brand. In this scenario, firms benefit from minimal spillover. In the other extreme, if brand spillover is sufficiently large, a joint branding regime is optimal (even in the absence of branding costs) as well because of the boost the lower-tier product receives. In this scenario, firms benefit from substantial spillover by foregoing profits of its premium product in order to sell higher amounts of the lower-tier product at a more favorable price. To summarize, we find that joint branding is optimal under the presence of small or substantial spillover but not in between. If the spillover is moderate, the boost received by the lower-tier product does not outweigh the loss incurred by the premium product.

What is more, we find that a larger fraction of sophisticated consumers does not necessarily push the firm towards joint branding, despite their indifference among branding regimes and the cost of erecting an additional brand. This result stems from the firm's inability to cater its prices to either consumer type. As a consequence, profits under joint branding are convex in the fraction of sophisticated consumers. Additionally, we establish that the branding decision itself is more than just a call under which name to market. It is in fact a decision about product lines as well. We show that substantial product confusion may push a firm to sell an inefficient premium product which it would not sell under a separate branding regime. The only reason to sell such a "halo product", even at a loss, is its beneficial effect, its externality, on the lower-tier product. We also find that firms are more likely to jointly brand more similar products. Moreover, when branding jointly, they decrease the price dispersion between their products.

Competition of firms sporting similar vertically differentiated product lines results in asymmetric equilibria. Firms that are not differentiated well from each other in terms of their products, have an incentive to do so via branding. This, however, is only feasible if branding effects are sufficiently high. Low-end (high-end) competition pushes the firm towards joint branding. While a monopolist may opt for joint branding to solve its in-house cannibalization problem, the same firm draws an additional benefit in the presence of low-end (high-end) competition from such a regime. Joint

branding allows the firm to differentiate itself better from its low-end rivals. Also, a firm selling vertically differentiated may be willing to provide consumers with information about the quality of its products—even if doing so is costly—when product confusion is limited. Under considerable product confusion, however, i.e., when the accuracy of available information is low, firms prefer to uphold the obfuscation due to joint branding.

What can our model can say about the likely consequences of the ongoing digitization of the economy? As more products continue to move online, it appears that the enhanced accuracy of available information decreases product confusion. Unless product confusion was substantial in the first place, this suggests that the growth of online retailing pushes firms towards joint branding. This is noteworthy as it establishes that more accurate information may lead firms to obfuscate more often. This result may reverse for products for which showrooming is essential, i.e., whenever the inability of consumers to physically inspect a products leads to significant product confusion. As a result, firms may be pushed to brand such products separately or provide free customer returns.

While we treat product quality as a fixed entity throughout this paper, firms' decisions about product characteristics and branding in real-world markets are often intertwined. Building a model with exogenous product quality serves two important purposes, however. First, it fosters insight about numerous scenarios where product quality is chosen from a discrete rather than from a continuous set. Levels of feasible product quality may for example be coarse when pre-determined by regulation or technology. Even more importantly, it allows us to to analyze the optimal branding regime for all possible combinations of vertical differentiated product lines in a general way.[16] Nevertheless, we believe that endogenizing product quality when considering a more narrow application to be a fruitful direction for future research.

Finally, we do not mean to invalidate the numerous other perils and promises of joint branding described in the literature, but rather urge both practitioners as well as researchers to consider the consequences of product confusion when making essential business, or research design decisions.

[16] An inspection of our model, in fact, establishes that—when choosing product quality—there are necessarily cases in which firms choose to offer only one product, and cases in which they would optimally offer two products, sometimes branded jointly and sometimes not. This can be seen when varying both marginal production cost as a function of product quality and the extent of product confusion.

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Appendix

Proof of Lemma 1. The perceived qualities follow Equation 3 when $k = s$. Hence, they are the same under any branding regime and the equilibrium prices and demands will be identical whether the firm adopts separate or joint branding. The latter, however, saves the firm on branding fixed costs ($\Delta > 0$) and hence will dominate. □

Proof of Lemma 2. To maximize their utility in (1), consumers self select into one of three purchase strategies as follows:

$$\begin{cases} \text{Buy product } h & \text{if } \max\left\{\frac{p_h-p_l}{\hat{q}_h-\hat{q}_l}, \frac{p_h}{\hat{q}_h}\right\} \leq \theta \leq 1 \Rightarrow u_h \max\{u_l, 0\} \\ \text{Buy product } l & \text{if } \frac{p_l}{\hat{q}_l} \leq \theta < \min\left\{\frac{p_h-p_l}{\hat{q}_h-\hat{q}_l}, 1\right\} \Rightarrow u_l \geq \max\{u_h, 0\} \\ \text{Buy neither product} & \text{if } 0 \leq \theta < \min\left\{\frac{p_l}{\hat{q}_l}, \frac{p_h}{\hat{q}_h}\right\} \Rightarrow 0 > \max\{u_h, u_l\}. \end{cases}$$

By simple algebraic manipulation, we have $p_h \leq \frac{\hat{q}_h}{\hat{q}_l} p_l \Leftrightarrow \frac{p_h-p_l}{\hat{q}_h-\hat{q}_l} < \frac{p_l}{\hat{q}_l}$, and no consumers buy l. In contrast, $p_l + \hat{q}_h - \hat{q}_l \leq p_h \Leftrightarrow 1 < \frac{p_h-p_l}{\hat{q}_h-\hat{q}_l}$, and no consumers buy h . It follows that $\frac{\hat{q}_h}{\hat{q}_l} p_l \leq p_h \leq p_l + \hat{q}_h - \hat{q}_l \Leftrightarrow \frac{p_l}{\hat{q}_l} \leq \frac{p_h-p_l}{\hat{q}_h-\hat{q}_l} \leq 1$, and both products get positive demand as per Equation 4.

The equilibrium prices in (5) are derived using respective first order conditions such that

$$\left(p^*_{h,SB}, p_{l,SB*}\right) = \left(\frac{q_h + c_h}{2}, \frac{q_l}{2}\right) = \arg\max_{p_h, p_l} \left[\left(p_h - c_h\right) x_{h,SB} + p_l\, x_{l,SB}\right],$$

where perceived qualities follow Equation 3 when $b = SB$ and $k = n$; and the resulting demand functions follow (4). The remaining results of the Lemma directly follow. □

Proof of Lemma 3. Under joint branding, perceived qualities follow Equation 3 when $b = JB$ and $k = n$; and the resulting demand functions follow (4). The firm chooses equilibrium prices to solve

$$\max_{p_h, p_l} \left[\left(p_h - c_h\right) x_{h,JB} + p_l\, x_{l,JB}\right]$$

Solving the FOC's yields the equilibrium prices and demand in Equation 7:

$$p^*_{h,JB} = p^*_{h,SB} - t_p(\beta),\ p^*_{l,JB} = p^*_{l,SB} + t_p(\beta),\ x^*_{h,JB} = x^*_{h,SB} - z_x(\beta),\ \text{and } x^*_{l,JB} = x^*_{l,SB} + z_x(\beta).$$

By direct substitution, $x^*_{h,JB} \geq 0 \Leftrightarrow \beta \leq \overline{\beta}_n \equiv \dfrac{\lambda - 1}{2\lambda}$ from Condition 1′. The remaining results of the Lemma directly follow. □

Proof of Corollary 1. From the results of Lemma 3, it follows that, for all $\beta \in [0, \overline{\beta}_n]$,

$$\pi^*_{JB} = \pi^*_{SB} - \frac{\beta\, c_h}{4\lambda}\left(\lambda^2 - \frac{2}{1-2\beta}\right) \qquad \Rightarrow \qquad \frac{\partial \pi^*_{JB}}{\partial \beta} = \frac{c_h}{4\lambda}\left(\lambda^2 - \frac{2}{(1-2\beta)^2}\right) \tag{13}$$

Therefore, π^*_{JB} is decreasing in β *if and only if* $(\lambda,\beta) \in [\sqrt{2},\infty] \times \left[0, \dfrac{\lambda-\sqrt{2}}{2\lambda}\right]$; otherwise, it is increasing in β. □

Proof of Proposition 1. When $\Delta = 0$, the firm chooses branding regime: $b = JB \Leftrightarrow \pi^*_{JB} > \pi^*_{SB} \Leftrightarrow \dfrac{\beta\, c_h}{4\lambda}\left(\dfrac{2}{1-2\beta} - \lambda^2\right) > 0$, which reduces to

$$\beta > \beta_2|_{\Delta=0} = \frac{\lambda^2-2}{2\lambda^2}.$$

First, $\beta_2|_{\Delta=0} < 0$ whenever $\lambda \in \Lambda_l \equiv \left[1,\sqrt{2}\right)$, and joint branding is dominant for all $\beta \in \beta \in [0,\overline{\beta}_n]$. Second, $\beta_2|_{\Delta=0} \in [0,\overline{\beta}_n]$. whenever $\lambda \in \Lambda_m \equiv \left[\sqrt{2},2\right)$, and joint (separate) branding is dominant if β is above (below) $\beta_2|_{\Delta=0}$. Third, $\beta_2|_{\Delta=0} > \overline{\beta}_n$ whenever $\lambda \in \Lambda_h \equiv [2,\infty)$, and separate branding is dominant for all $\beta \in \beta \in [0,\overline{\beta}_n]$. □

Proof of Proposition 2. Define $Z(\lambda,\beta,\Delta) \equiv \pi^*_{JB}(\lambda,\beta) - (\pi^*_{SB}(\lambda) - \Delta)$. From (13),

$$Z(\lambda,\beta,\Delta) = \Delta + \frac{\beta\, c_h}{4\lambda}\left(\frac{2}{1-2\beta} - \lambda^2\right).$$

Let $\beta = \beta_1(\lambda,\Delta)$, and $\beta = \beta_2(\lambda,\Delta)$ be the 2 roots that solve $Z(\lambda,\beta,\Delta) = 0$, and let for all $\lambda \geq \lambda_1(\Delta)$, such that

$$\beta_1(\lambda,\Delta) = \frac{8\Delta\lambda + c_h\left(\lambda^2-2\right) - \sqrt{A}}{4\lambda^2 c_h}, \text{ and } \quad \beta_2(\lambda,\Delta) = \frac{8\Delta\lambda + c_h\left(\lambda^2-2\right) + \sqrt{A}}{4\lambda^2 c_h} \tag{14}$$

where

$$\begin{aligned}
&A(\lambda,\Delta) \equiv (8\Delta\lambda + c_h(\lambda^2-2))^2 - 32\Delta\lambda^3 c_h,\\
&\lambda_1(\Delta) = \sqrt{2} + \frac{4\Delta + 2\sqrt{2\Delta\left(\sqrt{2}c_h + 2\Delta\right)}}{c_h}: \qquad A(\lambda_1,\Delta) = 0 \Rightarrow \beta_1(\lambda_1,\Delta) = \beta_2(\lambda_1,\Delta),\\
&\lambda_2(\Delta) = \frac{1}{2}\left(\frac{8\Delta}{c_h} + \sqrt{\left(\frac{8\Delta}{c_h}+3\right)^2 - 8} + 3\right): \quad \beta_2(\lambda_2,\Delta) = \overline{\beta}_n.
\end{aligned} \tag{15}$$

Therefore, no roots exist within $[0,\overline{\beta}_n]$ whenever $\lambda \in [1,\lambda_1)$; and the firm always chooses joint branding, for all $\beta \in [0,\overline{\beta}_n]$. Two real roots exist ($\beta_1$, and β_2 such that $0 \leq \beta_1 \leq \beta_2 \leq \overline{\beta}_n$) whenever $\lambda \in [\lambda_1,\lambda_2]$; and the firm chooses separate branding if $\beta \in [\beta_1,\beta_2]$, otherwise, it chooses joint branding. Finally, Only one real root (β_1) exists within $[0,\overline{\beta}_n]$ whenever $\lambda_2 < \lambda$; and the firm chooses separate branding if $\beta \leq \beta_1$, otherwise, it chooses joint branding. Setting $\Delta = 0$ yields the results of Proposition 1, where $\Lambda_l = [1,\lambda_1(0))$, $\Lambda_m = [\lambda_1(0),\lambda_2(0))$, $\Lambda_h = [\lambda_2(0),\infty]$, and $\beta_2|_{\Delta=0} = \beta_2(\lambda,0)$. □

Proof of Lemma 4. Under joint branding, perceived qualities follow Equation 3 when $b = JB$ and

$k = n$; and the resulting demand functions follow (4). The firm chooses equilibrium prices to solve

$$\max_{p_h, p_l} \left[(p_h - c_h)\, x_{h,JB} + p_l\, x_{l,JB} \right]$$

Solving the FOC's yields the equilibrium prices and demand in Equation 10. By direct substitution, the naive segment generates positive demand if and only if $\beta \leq \overline{\beta}_{mix}$ from Condition 1. The remaining results of the Lemma directly follow. □

Proof of Proposition 3. Define $Y(\alpha;\beta) \equiv \pi^*_{JB}(\alpha,\beta) - \pi^*_{SB}(\alpha)$, such that the firm choses joint (separate) branding if Y is above (below) $-\Delta$. Substituting demands and prices from Lemma 4, we first establish that Y is strictly convex in α:

$$\begin{aligned}
Y(\alpha;\beta) &= \frac{(1-\alpha)\beta\left(2(1-2\alpha\beta)c_h^2\left(\alpha\beta q_h + (1-\alpha\beta)q_l\right) - (1-2\beta)\left(q_h - q_l\right)^2\left(2\alpha\beta q_h + q_l\right)\right)}{4(1-2\beta)(1-2\alpha\beta)\left(q_h - q_l\right)\left(\alpha\beta q_h + (1-\alpha\beta)q_l\right)}, \\
Y''(\alpha;\beta) &= X_1\left(X_2 + q_l\left(X_4\beta\left(q_h^2 - q_l^2\right) + X_3 q_l^2\right)\right), \text{ where} \\
X_1(\alpha;\beta) &\equiv \frac{\beta^2\left(q_h - q_l\right)}{2(1-2\alpha\beta)^3\left(\alpha\beta q_h + (1-\alpha\beta)q_l\right)^3} > 0, \\
X_2(\alpha;\beta) &\equiv (1-2\beta)\left(2\alpha\beta\left(2\alpha^2\beta^2 + 3(1-2\alpha\beta)\right) + 1\right) q_h q_l^2 + 4\alpha^3\beta^3(1-2\beta)q_h^3 > 0, \\
X_3(\alpha;\beta) &\equiv (1-2\beta)\left(4\alpha^2\beta^2(3-2\alpha\beta) + 3(1-2\alpha\beta)\right) > 0, \text{ and} \\
X_4(\alpha;\beta) &\equiv 1 + 2\alpha\beta\left(2\alpha\left(4\alpha\beta^2 - 3(\alpha+1)\beta + 3\right) - 3\right).
\end{aligned}$$

Above, X_1, X_2, and X_3 are unambiguously positive for all $(\alpha,\beta) \in [0,1] \times [0,\frac{1}{2}]$. Note that $X_4'' = 24\beta(1-\beta-\beta(3-4\beta)\alpha)$. Hence, $X_4'' > 0$, for all $\alpha \in [0,\hat{\alpha}]$, where $\hat{\alpha} \equiv 1 + \frac{1-4\beta(1-\beta)}{\beta(3-4\beta)} > 1$ because β is bounded above by $\frac{1}{2}$. This establishes that X_4 is strictly convex in α for all $(\alpha,\beta) \in [0,1]\times[0,\frac{1}{2}]$. Solving the FOC yields $\alpha^* \equiv \arg\min_{\alpha} X_4 = \dfrac{1}{2(1-\beta) + \sqrt{2(1-2\beta)(2-3\beta)}}$. By direct substitution, $\min_{\alpha} X_4 = X_4(\alpha^*;\beta) = \dfrac{2(1-2\beta)\left(34\beta^2 - 46\beta + 16 + (8-9\beta)\sqrt{2(1-2\beta)(2-3\beta)}\right)}{\left(2(1-\beta) + \sqrt{2(1-2\beta)(2-3\beta)}\right)^3} > 0$. Thus, $X_4 > 0$ and $Y'' > 0$ for all $(\alpha,\beta) \in [0,1] \times [0,\frac{1}{2}]$.

Next, we turn attention to the first derivative, Y'. Recall that $\lambda \equiv \frac{q_h - q_l}{c_h} \geq \sqrt{2}$ by Condition 2.

$$\begin{aligned}
Y'(\alpha;\beta) &= \frac{\beta}{4}\left(\frac{\left(q_h - q_l\right)\left(2\alpha^2\beta^2(1-2\beta)q_h^2 - \beta(1-4\alpha(1-\beta)(1-\alpha\beta))q_h q_l + (1-\beta(3-2\alpha(2-\alpha)\beta))q_l^2\right)}{(1-2\alpha\beta)^2\left(\alpha\beta q_h - \alpha\beta q_l + q_l\right)^2}\right. \\
&\quad \left. - \frac{2c_h^2}{(1-2\beta)\left(q_h - q_l\right)}\right), \\
Y'(1;\beta) &\equiv \left.\frac{\partial Y}{\partial \alpha}\right|_{\alpha=1} = \frac{\beta\, c_h^2\left(\lambda^2 - 2\right)}{4(1-2\beta)\left(q_h - q_l\right)} + \frac{\beta^2\left(q_h - q_l\right)\left(q_h + q_l\right)}{4(1-2\beta)\left(\beta q_h + (1-\beta)q_l\right)} > 0, \forall \beta \in [0,\frac{1}{2}], \text{ and}
\end{aligned}$$

$$Y'(0;\beta) = \frac{1}{4}\beta\left((1-2\beta)q_h - \frac{2c_h^2}{(1-2\beta)(q_h-q_l)} - \frac{\beta q_h^2}{q_l} - (1-3\beta)q_l\right) \Rightarrow \quad Y'(0;\beta)\begin{cases} > 0 & \text{if } \beta < \hat{\beta} \\ < 0 & \text{if } \beta > \hat{\beta}, \end{cases}$$

where $\hat{\beta} \equiv \dfrac{(q_h-q_l)(q_h+5q_l) - \sqrt{16c_h^2 q_h q_l + 2q_l^2\left(24c_h^2 - q_h^2\right) + q_h^4 + q_l^4}}{4(q_h-q_l)(q_h+3q_l)}$, such that $\Delta \in [0, \overline{\Delta}) \Rightarrow \beta_1 < \hat{\beta}$, where β_1 and $\overline{\Delta}$ are given in Equation 14 and Condition 3, respectively.

Thus, $Y'(0;\beta) > 0$, for all $\beta \in [0, \beta_1)$; and because Y is strictly convex, this directly implies that Y is monotonically increasing in α , i.e., $Y'(\alpha;\beta) > 0$, for all $(\alpha, \beta) \in [0,1] \times [0, \beta_1)$. From Proposition 2, we also have $Y(0,\beta) > -\Delta$, for all $\beta \in [0, \beta_1)$. Since Y is monotonically increasing in α in this β range , we must have $Y(\alpha,\beta) > -\Delta$, for all $(\alpha, \beta) \in [0,1] \times [0, \beta_1)$, which concludes the proof of part (i). See Figure 8(a) for illustration.

From Proposition 2, we have $Y(0,\beta) < -\Delta < 0$, for all $\beta \in (\beta_1, \beta_2)$. Also, $Y(1,\beta) = 0 > -\Delta$. Since Y is strictly convex, it must cross $-\Delta$ exactly once: there exists a unique $\alpha_2 \in [0,1]$ that solves $Y(\alpha_2;\beta) = -\Delta$, for all $\beta \in (\beta_1, \beta_2)$; and the interval $A_h \equiv [\alpha_2, 1]$. This concludes the proof of part (ii). See Figure 8(b) for illustration.

From Proposition 2, we have $Y(0,\beta) > -\Delta$, for all $\beta \in (\beta_2, \overline{\beta}_{mix}]$. Also, $Y(1,\beta) = 0 > -\Delta$. Since Y is strictly convex, it must cross $-\Delta$ exactly twice: only two values, α_1, $\alpha_2 \in [0,1]$ satisfy $Y(\alpha_1;\beta) = Y(\alpha_2;\beta) = -\Delta$, for all $\beta \in (\beta_2, \overline{\beta}_{mix}]$; and the intervals $A_l \equiv [0, \alpha_1)$, $A_m \equiv [\alpha_1, \alpha_2)$, and $A_h \equiv [\alpha_2, 1]$. This concludes the proof of part (iii). See Figure 8(c) for illustration. □

Proof of Proposition 4. The firm providing full information is equivalent to the case where $\alpha = 1$. Hence, separate branding cannot arise as an equilibrium outcome because the firm can always achieve the same profits with joint branding and providing full information without the additional fixed cost (see Lemma 1). Thus, the firm will choose to provide no information if and only if $Y(\alpha;\beta) \equiv \pi^*_{JB}(\alpha,\beta) - \pi^*_{SB}(\alpha) > 0$. From Proposition 1, we have $Y(0;\beta) > 0 \;\Leftrightarrow\; \beta \in (\beta_2|_{\Delta=0}, \overline{\beta}_{mix}]$. The proof of Proposition 3 above established that Y is strictly convex and that $Y(1;\beta) = 0$. Thus, there must exist a unique $\hat{\alpha}_1 \in (0,1)]$ that solves $Y(\hat{\alpha}_1;\beta) = 0$. The results directly follow. □

Proof of Proposition 5. When two firms with similar product lines adopt asymmetric branding strategies, their demand functions are

$$x_{h,SB} = 1 - \frac{p_{h,SB} - p_{h,JB}}{\beta(q_h - q_l)}, \qquad x_{h,JB} = \frac{p_{h,SB} - p_{h,JB} - \beta(2p_{h,SB} - p_{h,JB} - p_{l,JB})}{\beta(1-2\beta)(q_h-q_l)},$$
$$x_{l,SB} = \frac{p_{l,JB} - p_{l,SB}}{\beta(q_h-q_l)} - \frac{p_{l,SB}}{q_l}, \quad x_{l,JB} = \frac{\beta(p_{h,JB} + p_{l,JB} - 2p_{l,SB}) - (p_{l,JB} - p_{l,SB})}{\beta(1-2\beta)(q_h-q_l)}.$$

Simultaneously solving the four FOC's from the firms' profit maximization problems yields the following equilibrium prices and quantities:

$$
\begin{aligned}
p^*_{h,SB} &= \frac{(2\beta\,(q_h-q_l)+3c_h)\,(4\beta q_h+3(1-\beta)q_l)}{4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l},\\
p^*_{h,JB} &= \frac{c_h\left(4\beta(3-\beta)q_h+(3-2\beta)^2q_l\right)+\beta\,(q_h-q_l)\left(4(1-\beta)\beta q_h+\left(4\beta^2-6\beta+3\right)q_l\right)}{4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l},\\
p^*_{l,SB} &= \frac{\beta q_l\,(2\beta\,(q_h-q_l)+3c_h)}{4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l},\\
p^*_{l,JB} &= \frac{2\beta\,(2\beta\,(q_h-q_l)+3c_h)\,(\beta q_h+(1-\beta)q_l)}{4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l},\\
x^*_{h,SB} &= \frac{2\beta\,(q_h-q_l)\,(4q_h-3q_l-2c_h)+3q_l\,(2\,(q_h-q_l)-c_h)}{(q_h-q_l)\,(4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l)},\\
x^*_{h,JB} &= \frac{(1-2\beta)\,(q_h-q_l)\,(4\beta q_h+3(1-\beta)q_l)-2c_h\,(\beta(4-\beta)\,(q_h-q_l)+3q_l)}{(1-2\beta)\,(q_h-q_l)\,(4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l)},\\
x^*_{l,SB} &= \frac{(2\beta\,(q_h-q_l)+3c_h)\,(\beta q_h+(1-\beta)q_l)}{(q_h-q_l)\,(4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l)},\\
x^*_{l,JB} &= \frac{\beta(1-2\beta)q_l\,(q_h-q_l)+c_h\,(2\beta(\beta+3)\,(q_h-q_l)+6q_l)}{(1-2\beta)\,(q_h-q_l)\,(4\beta(3+\beta)q_h+(9-6\beta-4\beta^2)\,q_l)}.
\end{aligned}
$$

The equilibrium profits are $\pi^*_{JB}=(p^*_{h,JB}-c_h)x^*_{h,JB}+p^*_{l,JB}x^*_{l,JB}$ and $\pi^*_{SB}=(p^*_{h,SB}-c_h)x^*_{h,SB}+p^*_{l,SB}x^*_{l,SB}$. At the limit as $\beta\to 0$, all prices and profits converge to marginal costs and zero, respectively. Evaluating the derivative of profits w.r.t. to β yields

$$
\left.\frac{\partial\pi^*_{JB}}{\partial\beta}\right|_{\beta=0}=\frac{c_h\left(\lambda^2-4\lambda+8\right)}{9\lambda}>0,\ \text{and}\quad \left.\frac{\partial\pi^*_{SB}}{\partial\beta}\right|_{\beta=0}=\frac{2c_h\left(2\lambda^2-2\lambda+1\right)}{9\lambda}>0,\quad \forall\lambda\in\mathbb{R}.
$$

Therefore, for sufficiently small fixed branding costs, their must exist a $\tilde{\beta}_{JB}>0$ such that the joint branding firm's profits surpasses its fixed branding cost whenever $\beta>\tilde{\beta}_{JB}$. Similarly, there must a $\tilde{\beta}_{SB}>0$ such that the separate branding firm's profits surpasses its fixed branding cost whenever $\beta>\tilde{\beta}_{SB}$. Let $\tilde{\beta}\equiv\max\{\tilde{\beta}_{JB},\tilde{\beta}_{SB}\}$. If the firms adopt similar brand strategies, their profits go to zero. If the firms adopt asymmetric brand strategies and $\beta<\tilde{\beta}$, the firms achieve positive profits but at least one firm cannot re-coupe its branding fixed costs. If the firms adopt asymmetric brand strategies and $\beta>\tilde{\beta}$, both firms achieve positive profits above their respective branding fixed costs. □

Proof of Lemma 5. If the firm chooses separate branding and sells both products, it's profits will be $\pi_2=\frac{1}{4}\left(\frac{q_e^2(q_h-q_l)-8q_eq_l(q_h+q_l)+16q_hq_l^2}{(4q_l-q_e)^2}-c_h\left(2-\frac{c_h}{q_h-q_l}\right)\right)$ (see proof of Lemma 6 below). If the firm chooses separate branding and drops product l, it's profits will be $\pi_1=\frac{(2q_h(q_h-q_e)-c_h(2q_h-q_e))^2}{(4q_h-q_e)^2(q_h-q_e)}$. Define $f_1(q_e)\equiv\frac{c_h^2\left(4q_l(2q_h-q_e)^2-q_e\left(8q_h^2-5q_eq_h+q_e^2\right)\right)}{(q_h-q_e)(q_h-q_l)}-q_e^2\left(\frac{(q_h-q_l)((8q_h+q_e)(q_l-q_e)+9q_l(8q_h-q_e))}{(4q_l-q_e)^2}+2c_h\right)$ and $f_2(q_e)\equiv\frac{1}{4(4q_h-q_e)^2}$ such that $\pi_2-\pi_1=f_1(q_e)\,f_2(q_e)$. Since $f_2(q_e)>0$, for all $q_e\in[0,q_l]$, then we

must have $\pi_2 - \pi_1 > 0 \Leftrightarrow f_1(q_e) > 0$. Using $\lambda \equiv \frac{q_h - q_l}{c - h}$ to simplify the exposition, we also have:

$$f_1(0) = \frac{16 c_h q_l (q_l + \lambda c_h)}{\lambda} > 0, \quad f_1(q_l) = -\frac{c_h q_l (\lambda + 1)(8\lambda(\lambda - 1)c_h + (7\lambda - 5)q_l)}{\lambda} < 0, \text{ and}$$

$$f_1'(q_e) = -\left(2c_h^2 \left(\frac{4q_h - q_e}{q_h - q_l} + \frac{2q_e(2q_h - q_e)}{(q_h - q_e)^2} \right) + 4c_h q_e \right.$$
$$\left. + \frac{2q_e(q_h - q_l)\left(q_e^3 + 4q_e^2(q_h - q_l) + 48q_l(q_h(q_l - q_e) + q_l(q_h - q_e)) + 224 q_h q_l^2\right)}{(4q_l - q_e)^3} \right) < 0.$$

The unique $\overline{q}_e \in [0, q_l]$ satisfies $f_1(\overline{q}_e) = 0$, and the result directly follows. □

Proof of Lemma 6. The demand functions of the three products is given by

$$x_h^s = 1 - \frac{p_h - p_l}{\hat{q}_h^s - \hat{q}_l^s}, \qquad x_l^s = \frac{p_h - p_l}{\hat{q}_h^s - \hat{q}_l^s} - \frac{p_l - p_e}{\hat{q}_l^s - q_e}, \qquad \text{and} \qquad x_e^s = \frac{p_l - p_e}{\hat{q}_l^s - q_e} - \frac{p_e}{q_e}.$$

After the focal firm chooses its branding strategy, $s \in \{SB, JB\}$, the two firms simultaneously choose prices to optimize their respective profits. Solving FOC's simultaneously yields the following subgame equilibrium outcomes under each of the two branding subgames:

$$\begin{array}{lll}
p_{h,SB}^* = \frac{q_h + c_h}{2} - \frac{3q_e q_l}{2(4q_l - q_e)}, & p_{l,SB}^* = \frac{q_l}{2} - \frac{3q_e q_l}{2(4q_l - q_e)} & p_{e,SB}^* = \frac{q_e(q_l - q_e)}{4q_l - q_e}, \\
x_{h,SB}^* = \frac{1}{2}\left(1 - \frac{1}{\lambda}\right), & x_{l,SB}^* = \frac{1}{2} - x_{h,SB}^* + \frac{q_e}{2(4q_l - q_e)}, & x_{e,SB}^* = \frac{q_l}{4q_l - q_e}, \\
p_{h,JB}^* = p_{h,SB}^* - \left(t_p(\beta) - t_p^{comp}(\beta)\right), & p_{l,JB}^* = p_{l,SB}^* + \left(t_p(\beta) + t_p^{comp}(\beta)\right) & p_{e,SB}^* = p_{e,SB}^* + 2t_p^{comp}(\beta), \\
x_{h,JB}^* = x_{h,SB}^* - z_x(\beta), & x_{l,JB}^* = x_{l,SB}^* + \left(z_x(\beta) - z_x^{comp}(\beta)\right), & x_{e,JB}^* = x_{e,SB}^* - \frac{z_x^{comp}(\beta)}{2},
\end{array} \tag{16}$$

where $z_x^{comp}(\beta) = \frac{2\beta q_e(q_h - q_l)}{(4q_l - q_e)(4q_l - q_e + 2\beta(q_h - q_l))}$ and $t_p^{comp}(\beta) = \frac{3}{4} q_e z_x^{comp}(\beta)$; and $t_p(\beta)$ and $z_x(\beta)$ are given in Equation 7. The result directly follows. □

Proof of Proposition 6. We will use the superscripts "*mon*" and "*comp*" to differentiate the profits under monopoly and competition. From Equations 5 and 7 monopoly profits under SB and JB are

$$\pi_{mon}^{SB*} = \frac{1}{4}\left[q_h - c_h\left(2 - \frac{c_h}{q_h - q_l}\right)\right], \qquad \pi_{mon}^{JB*} = \pi_{mon}^{SB*} - A^{mon},$$

where $A^{mon} \equiv \frac{\beta c_h}{4\lambda}\left(\lambda^2 - \frac{2}{1-\beta}\right)$. The profits under competition can be derived from (16), such that

$$\pi_{comp}^{SB*} = \pi_{mon}^{SB*} - \frac{q_e q_l (q_e + 8q_l)}{4(4q_l - q_e)^2}, \qquad \pi_{comp}^{JB*} = \pi_{comp}^{SB*} - A^{mon} + A^{comp},$$

where $A^{comp} \equiv \frac{\beta\lambda c_h q_e^2(4\beta\lambda c_h(10q_l - q_e) + (4q_l - q_e)(20q_l + q_e))}{4(4q_l - q_e)^2(2\beta\lambda c_h - q_e + 4q_l)^2} > 0 \Rightarrow \frac{\partial A^{comp}}{\partial \beta} = \frac{\lambda c_h q_e^2(20\beta c_h\lambda + q_e + 20q_l)}{8(2\beta c_h\lambda - q_e + 4q_l)^3} > 0$. Therefore,

$$\left[\pi_{comp}^{JB*} - \pi_{comp}^{SB*}\right] - \left[\pi_{mon}^{JB*} - \pi_{mon}^{SB*}\right] = A^{comp} > 0$$

□

Proof of Corollary 2.

$$\frac{\partial\left[\pi_{comp}^{JB*}-\pi_{comp}^{SB*}\right]}{\partial q_e}=\frac{16\beta(q_h-q_l)(A_1+A_2+A_3+A_4+A_5)}{A_1} > 0, \text{ where}$$

$$A_1 \equiv 8(4q_l-q_e)^4(4q_l-q_e+4\beta(q_h-q_l))^4 > 0,$$

$$A_2 \equiv 4q_l(4q_l-q_e)^3\left(2q_e^2+23q_eq_l+20q_l^2\right) > 0,$$

$$A_3 \equiv 4\beta(4q_l-q_e)^2\left(240q_l^3-q_e^3+q_eq_l(200q_l-7q_e)\right)(q_h-q_l) > 0,$$

$$A_4 \equiv 4\beta^2(4q_l-q_e)\left(5q_e^3+q_eq_l(496q_l-60q_e)+960q_l^3\right)(q_h-q_l)^2 > 0, \text{ and}$$

$$A_5 \equiv 1024\beta^3q_l^2(q_e+5q_l)(q_h-q_l)^3 > 0.$$

□