Non-cooperative Bargaining and Collusion Formation Through Communication Networks[1]

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2024-8-22
GAMES 2024

Motivation

- Motivation: In many organizations, when enough people collude, they can cover each other up to avoid punishment.
- Example: A high-tech firm. A boss invested money. A group of engineers develop a secret technology.
- The product turned out to be a success. The boss takes all the profit r.
- One of engineers realize that he could copy the technology and start his own firm.
- Suppose, If m of the engineers agree, they can start the new firm and split the profit r among themselves, not necessarily equally.
- If m engineers move to the new firm, the old firm will go bankrupt.

Motivation

- However, this initiator (engineer) does not directly know everyone else in the firm, so he has to persuade his friends first.
- Once his friends agree, then they can connect this initiator with the others, until this threshold m is reached.
- Anticipating this, the boss also promises to give rewards to those who refuse to join the collusion and punish those who join.
- But, those rewards and punishments are only effective when the collusion fails.

Research Questions

- Since the boss takes away most of the benefits: Then why don't the engineers always start their own firm (collude)?
- How can the boss stop such collusion?
- What is the role of communication in this organizing collusion?
- Who are the key players in initiating the collusion?

Preview of Results

1. Less communication makes collusion harder.
2. Principal sharing profit also makes collusion harder.
3. Define m as the success threshold of collusion,
4. Exists $\overline{m}$ (critical threshold of a network), such that for all $m \leq \overline{m}$ collusion can succeed; while for all $m > \overline{m}$ collusion always fails.
5. A general solution for any communication network is provided.
6. Three limiting cases are proved for benchmarks.

Additional applications

This model also applies to following the cases:

- Firm employees collude to hide other misconducts.
- Government officials collude and cover each other's corruption.
- Congressmen/Senators seek support from each other to pass bills and share benefits.
- Organization of other collective actions: labor union, social unrest, regime change, revolution...

Literature

The structure of literature review is as follows.

1. Sequential Voting process, similar to (Rubinstein, 1982a; Binmore et al., 1986; Chatterjee et al., 1993; Ray, 2007; Battaglini, 2021).
   I follow their sequential choice model.
2. Resource distribution game. "Lenin vs. Czar" (Roemer, 1985; Grossman, 1991; Little, 2016; Enikolopov et al., 2011)
   Network structures was not modeled
3. Non-cooperative bargaining. (Rubinstein, 1982b; Britz et al., 2010; Ray and Vohra, 1997; Chatterjee et al., 1993; Okada, 1996)
   I provide a unique angle to model the importance of communication network.
4. Cooperative network bargaining. (Feinberg, 1998; Dannenberg et al., 2020; Hyndman and Ray, 2007; Jackson and Zenou, 2015)
   This game cannot be adopted as a cooperative game.

Base Model

- Principal (owner): player 0; Agents (engineers): player 1 to n.
- An existing company will earn profit r.
- The owner can choose how to allocate this profit r.
- The agents start within an exogenously given communication network. Personal connections are bilateral (no direction).
- Let $\mathbf{C}$ be an $n \times n$ symmetric matrix that represents the communication network.
- If $\mathbf{C}_{i,j} = 1$, it means player i can directly send collusion invitation to player j.
- Otherwise, $\mathbf{C}_{i,j} = 0$

Base Model: Timing

- $\mathbf{C}$ and m are exogenous.
- The principal (she) commit to a distribution rule to reward rejecters and punish participants of collusion.
- A random agent (he) is chosen to be the initiator and he chooses whether to enter the network bargaining.
- If so, the initiator sequentially approach other agents and offer them a share of profit in the new firm.
    - If at least m agents join collusion, the collusion start and the profit is distributed according to initiator's rule.
    - If less than m agents joins collusion. Collusion fails. Profit is distributed as the principal's rule.
- If not enter bargaining stage, nothing happens.
- A small establishment cost $c > 0$ for new firm.

Base Model: First Stage

- A principal (owner) chooses an initial distribution of profit.
- Initial profit distribution for agent(employee) i gets if there is no bargaining stage.

$$r_0(i) \geq 0$$

- $\sum_{i \in I} r_0(i) \leq r$ (feasibility).
- $r_0(i) = 0$ for all $i \in I$: No bonus. Otherwise, with bonus.
- If an agent j is contacted by initiator i, yet he refuses to join the collusion then agent j will get a reward.

$$r_1(j|h(i,j)) \geq 0$$

- $h(i,j)$ is the history until j is reached.
- $\sum_{j \in I} r_1(j|h(i,j)) \leq r$ (feasibility).

Base Model: Second Stage

- If there is one agent who can benefit from collusion, he will become the initiator.
- If multiple agents can benefit, one of them is randomly chosen.

- If no one can benefit from collusion, the game ends and everyone gets $r_0(i)$.

Base Model: Third Stage

- Third Stage: Bargaining stage
- The initiator chooses a player who he directly connects to and proposes a collusion offer.
- If collusion is successful. The initiator i offers commit to reward colleague j for:

$$e_1(j|i) \geq 0$$

- Then the receiver decides whether to accept this offer.
  - If the receiver accept, then he also agrees automatically to connect the initiator to all of the receiver's contacts, so the initiator can propagate the communication.
  - If a receiver rejects the offer, he also refuses to introduce his contacts to the initiator.
- Then, the initiator chooses another player to make offer.

Base Model: Third Stage

- If $m-1$ other agents to join collusion. Then, collusion succeed. Everyone gets

$$e_1(j|i)$$

- If less than $m-1$ other agents join. Then, collusion fails. Everyone gets

$$r_1(j|h(j|i))$$

- In equilibrium:
  $e_1(j|i) = 0$ for non-participants of collusion.
  $r_1(j|h(j|i)) = 0$ for participants of collusion.

Main Result

Proposition 1

The Two Critical Thresholds.

- *Given a communication network, there is a critical threshold* $\overline{m}_{\text{no share}}$*, such that for all success threshold* $m > \overline{m}_{\text{no share}}$*, collusion does not occur, and the principal does not need to share any profit with the agents in the original firm* ($r_0(i) = 0$) *for all* $i \in \mathbf{I}$.
- *There is another critical threshold* $\overline{m}_{\text{share}} \leq \overline{m}_{\text{no share}}$*, such that for all* $m > \overline{m}_{\text{share}}$*, the collusion does not occur, but* $r_0(i) \geq 0$ *for some agents* $i \in \mathbf{I}$.
- *For* $m \leq \overline{m}_{\text{share}}$*, collusion always occurs and the principal gets zero payoff.*

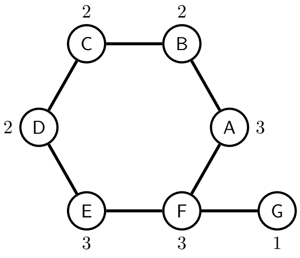
Results

- No Profit share algorithm.
- Profit sharing algorithm.
- Extreme Cases
  1. Complete network
  2. Ring
  3. Star

No Profit Share Algorithm (Intuition)

- The details are omitted for the sake of time. See appendix.
- First, for each $m \in 1, 2, 3, ..., n$, and for each $i \in N$, check all possible communication paths from i to reach $m-1$ accepters.
- Calculate if there is a $r_1(j|h(i|j))$, that satisfies the two conditions below:
- **(Blocking Condition)** the sum of $r_1(j|h(i|j))$ is larger than $r - c$. Because a necessary condition for receiver to join collusion is $e_(j|i) \geq r_1(j|h(i|j))$
- **(Feasibility Condition)** the principal's reward never not exceeds its resource constraint r.
- For each $i \in N$, find the lowest m such that an $r_1(j|h(i|j))$ exists.
- Individual critical threshold is one less than that number. Denote as $\overline{m}_{\textit{no share}}(i)$
- Network critical threshold $\overline{m}_{\textit{no share}} = \max_{i \in N} \overline{m}_{\textit{no share}}(i)$

No Profit Sharing: Examples



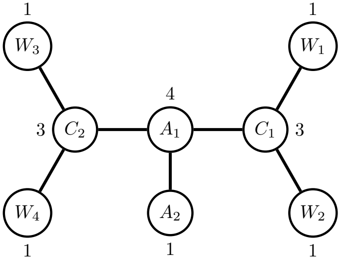


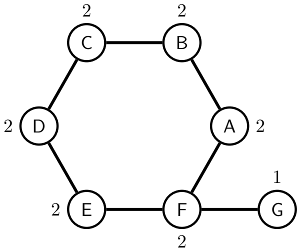
Figure: The figure on the left is a modified ring supervision network. The figure on the right is a tree supervision network. Agents are represented by letters in circles. The label next to each node represents the player's no share critical threshold $\overline{m}_{\textit{no share}}(i)$.

Algorithm

Profit Sharing Algorithm

- General procedure is the same.
- Sharing profit $r_0(i)$, so that player i has to reserve this amount to himself in the new equilibrium.
- Thus, fewer resources are left for the others.
- The complicated part is that giving one agent $r_0(i)$ will reduce the budget for the others. So the optimal allocation requires some additional steps.

Profit Sharing: Examples



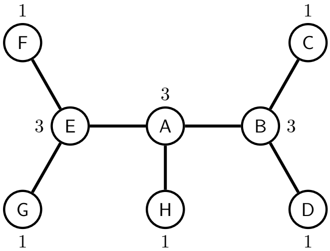


Figure: Agents are represented by letters in circles. The numbers next to each individual indicate the critical threshold with a share of each agent. The left figure's critical threshold with profit sharing is $\overline{m}_{share} = 2$. The right figure is the same tree as before. Its critical threshold with share dropped from 4 to $\overline{m}_{share} = 3$.

Comparative Statics

Proposition 2

For a given set of agents $\mathbf{N}$*, there is a communication network* $\mathbf{C}$*. Removing any communication link in* $\mathbf{C}$ *weakly decreases the critical threshold (with or without profit sharing) of all the agents.*

Proposition 3

For a given set of agents $\mathbf{N}$*, there is a communication network* $\mathbf{C}$*. In both cases with or without profit sharing:*

- *if the principal has more (less) resources to allocate, then the critical threshold of all the agents weakly decreases (increases).*
- *if the initiator has more (less) resources to allocate, then the critical threshold of all the agents weakly increase.*

Extreme Case: Complete Network

Proposition 4

When $n \geq 3$ and the communication network is complete. The principal can stop collusion if and only if

$$m > \frac{n^2 + n - 1}{2n - 1}. \tag{1}$$

In addition, collusion can be stopped even without profit sharing if and only if

$$m > \frac{n}{2} + 1 \tag{2}$$

Extreme Case: Ring/Line Network

Proposition 5

When the success threshold is $m = 3$ *and* $n \geq 3$ *and no profit sharing* $(r_0(i = 0)$ *for all* $i \in N$*), collusion can be stopped* **if and only if** *the communication network is a line or ring.*

Extreme Case: Star Network

Proposition 6

When the success threshold is $m = 2$ and $n \geq 3$, then the principal can stop collusion **if and only if** *the communication network is a star and the principal shares her profit with agents.*

Conclusion

- This paper focuses on collusion that cannot be punished if successful.
- We also study the network effect in the collusion formation process.
- A solution to general network is provided.
- Extreme cases are also proved.
- The model setting is flexible.
- Wide application to firms, government or even international politics.

Appendix

Results: The principal's equilibrium strategy

- The principal always choose $r_1(i|j)$ to deter collusion by j if he can do so without profit sharing.
- The principal may choose profit sharing $r_0(j) > 0$ to deter collusion if it is profitable to do so.
- The principal never choose $r_0(j) > 0$ if the cannot deter collusion initiated by j.
- If the principal cannot deter collusion initiated by j, then $r_1(i|j)$ can be any non-negative feasible number.

No Profit Share Algorithm

Definition 1

Blocking condition for initiator i. It is satisfied if and only if an initiator i cannot succeed in organizing the collusion for all communication paths that starts from i and contains m accepters $\sigma_i(m)$. Denote the set of such communication paths as $A_i(m)$

$$\sum_{j \in \sigma_i(m)} \mathbb{1}\{\text{j joins collusion}\} \cdot r_1(j|\sigma_{i,j} \sqsubseteq \sigma_i(m)) \geq r - c. \quad (3)$$

No Profit Share Algorithm

Definition 2

Feasibility Condition: this condition is satisfied if and only if for each $\sigma_i(m') \in A_i(m')$ that satisfies $m' \leq m$ and $\sum_{j \in \sigma_i(m')} r_1(j|\sigma_{i,j}) < r - c$, the following inequality holds:

$$\sum_{j \in \sigma_i(m')} \mathbb{1}\{\text{j not join collusion}\} \cdot r_1(j|\sigma_{i,j} \sqsubseteq \sigma_i(m)) \leq r. \quad (4)$$

No Profit Share Algorithm

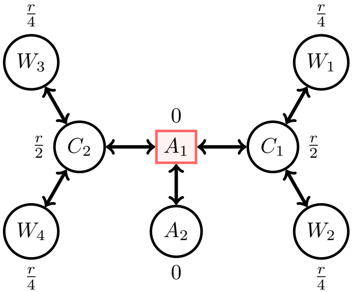


Figure: A tree network. Agents are represented by letters in circles, and the initiator, by letter A in a red square. Arrows point from supervisors to supervisees. The label next to each node is the corresponding transfer chosen by the principal.

No Profit Sharing, Algorithm

- For each initiator i, starting from $m = 1$, check if there is a vector $\{r_0(1), r_0(2), ..., r_0(n)\}$ such that for each initiator i, there exists a vector of $r_1(j|i)$ so that for each $\sigma_i(m) \in A_i(m)$, the following modified blocking condition holds.

$$\sum_{j \in \sigma_i(m)} r_1(j|i) \geq \sum_{j \neq i} r - r_0(i) \tag{5}$$

The right hand side is the disposable reward of the initiator i. It is the sum of the profit that goes to all the other players that has to be less than the total profit r minus what the initiator can get in the original equilibrium $r_0(i)$.

No Profit Sharing, Algorithm

- Then, check if there exists an $m' < m$, such that for all $\sigma_i(m') \in A_i(m')$, there exists a vector $\{r_1(1), r_1(2), ..., r_1(n)\}$ such that the following inequality feasibility constraint holds:

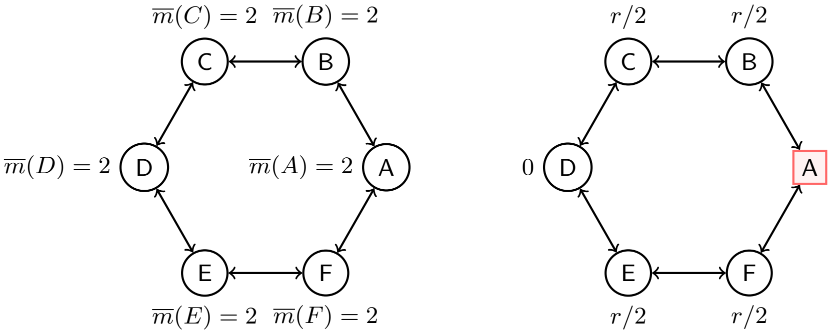
$$\sum_{j \in n(\sigma_i(m'))} r_1(j) \leq r. \tag{6}$$

- If both $\{r_0(1), r_0(2), ..., r_0(n)\}$ and $\{r_1(1), r_1(2), ..., r_1(n)\}$ exist, then the critical threshold is: $\overline{m}_{withshare} = m - 1$.
- If the solution does not exists, then increase m by 1 and repeat the first step.

Ring Network

Definition 3

r is the number of supervision links in the supervision network $\mathbf{S}$.
Here, r equals the total gain from collusion.



Figure

Star Network

- When If $n = 2$ and $m = 2$, the deviation can be stopped without profit sharing.
- When $m = 1$, deviation cannot be stopped for all $n \geq 2$

Networks $n = 2$ and $n = 1$

Proposition 7

- *When If $n = 2$ and $m = 2$, the deviation can be stopped without profit sharing.*
- *When $m = 1$, deviation cannot be stopped for all $n \geq 2$*

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