How to Make People Work Without Direct Supervision: A Network Characterization of Collusion Formation

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Motivation

- Peer supervision could be vulnerable to joint deviation to a new equilibrium while the principal cannot punish such a deviation.
- Real world examples of agents collude:
  - Government officials taking bribery: collude with the police or the court to avoid punishment.
  - Financial crime: agents jointly hide evidence.
  - Labor union formation: law protect union workers.
  - Insurrection or revolution: No punishment if successful.

This Paper

1. Model the collusion formation process as voting or commitment.
2. Central insight: Limiting the communication among the agents can effectively deter collusion.
3. The principal does not need to exert punishment or rewards.

Preview of Results

- Focus on the voting mechanism today.
- Exogenous success threshold: m agents say yes, then deviate to new equilibrium.
- Under fully connected supervision network, collusion can be stopped if and only if $m \geq \frac{n^2+n-1}{2n-1}$
- If putting the agents into a ring. For all $n \geq 3$, collusion can be stopped if $m \geq 3$.
- The ring is the most efficient and robust network that supports "full-effort".
- An algorithm to calculate the "bargaining power" of any agent in an arbitrary communication network.

Literature

- Akerlof (1976) first developed a model of community enforcement causing inefficient outcome.
- Monitoring or community enforcement (Kandori, 1992, Ellison, 1994, Kranton, 1996, Wolitzky, 2013, Ali and Miller, 2014, Dixit, 2011, Masten and Prüfer, 2014, Levine and Modica 2016, Aldashev and Zanarone, 2017, Acemoglu and Wolitzky, 2017, 2018, 2019).
    - **They focus on reinforcing efficient outcome while I find doing the opposite shall also be valuable.**
- Coalition formation and other refinement of Subgame perfect equilibrium. (Bernheim et al., 1987; Farrell and Maskin, 1989; Ray, 2007) .
    - **I build on their models and provide a new microfoundation for coalition formation process.**
- Protest and Revolution (Roemer, 1985; Enikolopov et al., 2011; Little, 2016; Basu, 2022): control communication to increase the cost of organizing collective action.
    - **I provide a different mechanism for communication cost.**

Base Model: No communication

- Play set: $\mathcal{N}$. principal: player 0; Agents: player 1 to n.
- At period 0, the principal choose an SPNE for the agents.
- First stage (morning) of period $t \geq 1$, agents choose effort and transfers.
- Second stage (afternoon), agents choose peer punishment.
- $e_{i,t} \in [0, \infty)$: effort level of player i.
- $\pi_{ijt} \in [0, \overline{\pi}]$: amount of transfers from i to j.
- $p_{ijt} \in [0, 1]$: punishment severity from i to j.
- A strategy s_i is a map from history to an action in the stage game. s_{-i} denotes the strategy set of the opponents.

Base Model: No communication

- The stage game utility function for an agent i:

$$u_{it}(\cdot) = -e_{it} - \sum_{j \neq i} \pi_{ijt} + \sum_{j \neq i} \pi_{jit} - \sum_{j \neq i} max\{p_{ijt}, p_{jit}\}$$

- The total utility for agent i:

$$U_i(s_i, s_{-i}) = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} u_{it}(s_i, s_{-i})$$

- The principal chooses an subgame perfect equilibrium *eq* for the agents to maximize the discounted total effort level of the agents.

$$U_p(eq) = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} \sum_{i \in I} e_{it}$$

- e_0: agent's effort level on the equilibrium path.

Base Model: No communication

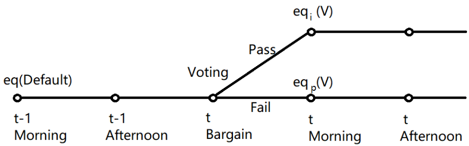
- *EQ*: the set of all subgame perfect equilibria.
- $u_{it} \geq -(n-1)$ is individual rational (by trigger strategy).
- $\overline{e_{it}} = (n-1)$: the full effort level.
- If the agents can communicate, they should jointly deviate to no effort ($e_{it} = 0$ for all i and t).
- The principal cannot punish the agents for insufficient effort, because the lack of evidence, or because the deviation is legal.

Model 2: Voting

- Thus, I want to model the renegotiation process.
- This paper models two ways of collusion formation.
  1. Vote for deviation. (Today's focus)
  2. Commitment for deviation.
- First: Introduce the voting process
- Second: Limit communication to improve robustness.

Model 2: Voting

- $eq_{default}$: The initial equilibrium chosen by the principal.
- Assume an exogenously given equilibrium selection rule: if an initiator proposes a deviation, and if at least m out of n agents vote yes, then everyone deviates, otherwise, all the players follows the principal's suggestion.
- p_0: probability that one agent becomes the initiator in each period.
- The initiator makes a proposal and calls the her colleagues one by one and asks them to vote.



Model 2: Voting

- $v_i \in \{yes, no\}$: individual vote.
- $v = \{v_1, v_2, ..., v_n\} \in V$: the set of individual votes.
- eq_i and eq_p are functions mapping from V to EQ.
  1. If at least m yes votes, then everyone follows eq_i.
  2. If less than m yes votes, then everyone follows eq_p.
- Assume that the principal can control the initial communication network.
- The initiator can "ask around for phone numbers".

Model 2: Voting

- Assume if player i can punish another player j, then they can directly communicate.
- $\mathbf{S}$: $n \times n$ matrix for the supervision network. (Fixed)
- $\mathbf{S}_{i,j} = 1$, if i is the supervisor of j (i can punish j). $s_{i,j} = 0$ otherwise.
- $\mathbf{C}$: $n \times n$ symmetric matrix for the communication network. (Changeable)
- $\mathbf{C}_{i,j} = 1$, if i can directly communicate with j. $\mathbf{C}_{i,j} = 0$ otherwise.
- If $\mathbf{S}_{i,j} = 1$ then $\mathbf{C}_{i,j} = \mathbf{C}_{j,i} = 1$.

Model 2: Voting

- No isolated agent in **C**.
- The bargaining stage ends if: m yes-votes are collected, or the initiator exhausts its contact list.
- The principal: chooses $eq_p(v)$ and **S**.
  1. Stop collusion if possible.
  2. Maximize the discounted sum of total effort.
- The initiator: chooses $eq_i(v)$ and the order of communication $\mathbf{O} = \{i_1, i_2, ..., i_{n-1}\}$, to maximize her utility.

Model 2: Voting

Proposition 1

Under fully connected communication network, the joint deviation to no effort can be stopped if and only if

$$m > \frac{n^2 + n - 1}{2n - 1} \tag{1}$$

Also, If $m > \frac{n}{2} + 1$*, agents can exert full effort* $e_0 = n - 1$ *in the default equilibrium, yet joint deviation does not occur.*

Voting: single ring

Proposition 2

Under deviation by voting (with a threshold m and department size n), the principal can stop joint deviation for all $m \geq 3$ and all $n \geq m$, using a single ring supervision network and a corresponding equilibrium selection function eq_p,

Corollary 1

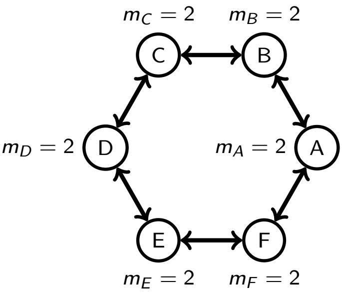
When $m = 2$ collusion can be stopped using a star network. However, full effort is not possible.

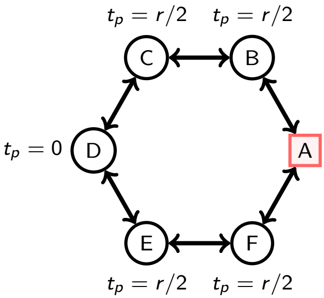
When $m = 1$ collusion cannot be stopped.

Voting: single ring

Definition 2

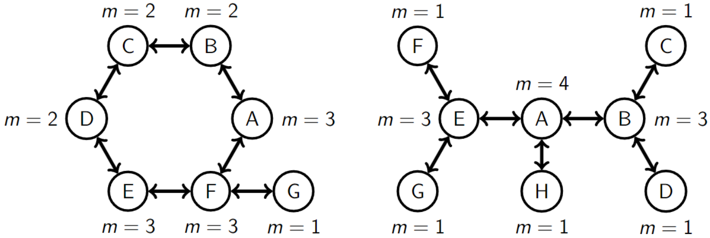
A single ring supervision network is one that $s_{i,i+1} = 1$, $s_{i+1,i} = 1$ for all $1 \leq i \leq n-1$, $s_{n,1} = s_{1,n} = 1$ and $s_{i,j} = 0$ otherwise. In other words, every player is both the supervisor and supervisee of her two neighbors.





Additional Results

- An algorithm to calculate the negotiation power of any individual in arbitrary network:
  - Negotiation power is max number players who will vote yes to the deviation plan. m_i
  - A new measure of centrality.



The number next to each node represents the player's negotiation power $m_i(S)$

Additional Results

- Deviation by commitment:
  - Allow the initiator to commit to arbitrary strategy.
  - Allow each receiver to join the commitment.
  - The single ring supervision network can stop deviation if there are at lest 6 agents.
- In conclusion: limiting the communication network and rewarding agents who reject the deviation proposal can effectively deter collusion.

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